

CONCEPTUAL
STARTERS
YEARS 7 - 8

PHASE 3

How to Guide

These conceptual starters have been designed and planned to meet learning intentions of the New Zealand curriculum. Whilst it is a large collection of starters there are many more starters that can be used in your mathematics programs.

Each starter is:

- Designed to be used more than once.
- Written with a small number of other examples, however almost all starters could be adapted and used with a variety more different numbers, patterns and materials.
- Encouraging the use of mathematical practices.
- Supporting the use of dialogue and communication during these starters.
- Designed to be chosen intentionally and used to revisit or build upon concepts taught throughout the year.

Mathematical Practices are:

Making an explanation

Arguing mathematically

Making a justification

Representing

Making a generalisation

Expect, scaffold, and support your students to use these mathematical practices when sharing their ideas during these starters.

Always, encourage and celebrate all contributions and ideas that are shared from all students. Be ambitious, don't limit your students to small numbers. In Phase Two they need exposure and chances to reason with numbers to at least 1,000,000.

Important Number Properties within this booklet:

- Inverse: division is the inverse operation of multiplication ($ax=b$ so $c \div b=a$). Multiplication facts give rise to families of facts that use division.
- Identity: when a number is added to 0 the result will be the same, when a number is multiplied or divided by 1 the result will be the same.
- Commutative: we can multiply or add two numbers in any order, and the sum will never change. This does not hold for division or subtraction. E.g., $a \times b = b \times a$, $a + b = b + a$.
- Associative: when adding or multiplying two or more numbers it does not matter what order they are added/multiplied in. E.g., $(a \times b) \times c = a \times (b \times c)$
- Distributive: each addend of a sum can be multiplied separately and the product will be the same (e.g., $3 \times 17 = 3 \times (10 + 7) = (3 \times 10) + (3 \times 7)$).

Most of these starters can be adapted and used as independent tasks as well.

All students can be successful mathematicians when given the opportunities to succeed.

Table of Contents

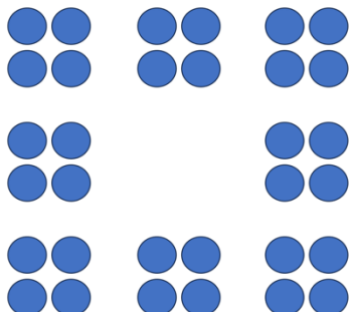
NUMBER : MĀTAURANGA TAU	6
Quick Images – Groups Of	6
Place-Value to 100,000,000	8
Reading and Explaining Numbers to 1,000,000,000	9
Choral Counting	10
How many 1's, 10's, 100's, 1000's	12
Order numbers to 100,000,000	13
Round to the nearest.....	14
Number Line.....	15
Multiplication Pyramid.....	16
Square numbers.....	17
Area Model.....	18
Divisibility.....	19
Division Strings – Partial Quotients.....	20
Multiplication Strings	22
Multiplication Strings – place value	24
True or false (powers)	25
True or false (adding powers)	26
If... Then.....	27
What's the Point? Positive and Negative Integers	28
What is the Story? Positive and Negative Integers	29
Fraction – Number line.....	30
True or False- Convert Improper Fractions	31
Fractions: Converting Improper Fractions (Number Line).....	32
Find the whole amount when given a percentage or fraction amount.	33
Fractional part of a set	34
Would you rather? (Fractions and percentages)	35
Mix and Match : Fractions, Decimals and Percentages	36
Fractions, Decimals, Percentages.....	37
Decimal Addition – Missing Addends.....	38
Multiply fractions and decimals by whole numbers.....	39
Percentages: Number String	40
Calculate costs and change for any amount of money.	41
Where is the maths?	42

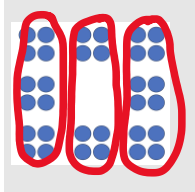
ALGEBRA / TAURANGI	43
True or False Statements with Symbols	43
True or False - integers.....	44
Balancing Equations	45
Balancing Equations (multiplication and division)	46
Solve simple equations with variables	47
True or False – Expanding Brackets.....	48
Function Machine	49
Algebraic Graphs.....	50
MEASUREMENT – INE	51
Conversions of measurements	51
Estimating Area and Perimeter.....	53
The perimeter iswhat is the Area?	54
EXPLORING AREA.....	55
EXPLORING AREA OF RIGHT ANGLE TRIANGLES.....	56
Calculating Volume	57
Scale factor – volume	59
Convert digital to analogue time	60
SPACE MOKOWĀ	61
Triangles.....	61
IS IT A QUADRILATERAL?.....	62
IS IT A POLYGON?.....	64
Is it a Prism?	66
Is this the correct net for this 3d shape?	67
Invariant properties	68
USING PLAN VIEWS.....	69
Geometry: What angles do you notice?	70
Intersecting Angles	71
Latitude and longitude	72
STATISTICS - TAUANGA	73
Bar Graph Starter.....	73
Graphs - What do you notice?	74
Stem and Leaf starter	75
Statistics Claim.....	76
PROBABILITY : TŪPONOTANGA	77

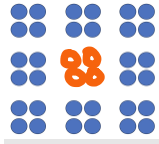
Probability – continuum	77
Theoretical probability verses experimental probability	78
Comparing Results	79

NUMBER : MĀTAURANGA TAU

QUICK IMAGES – GROUPS OF



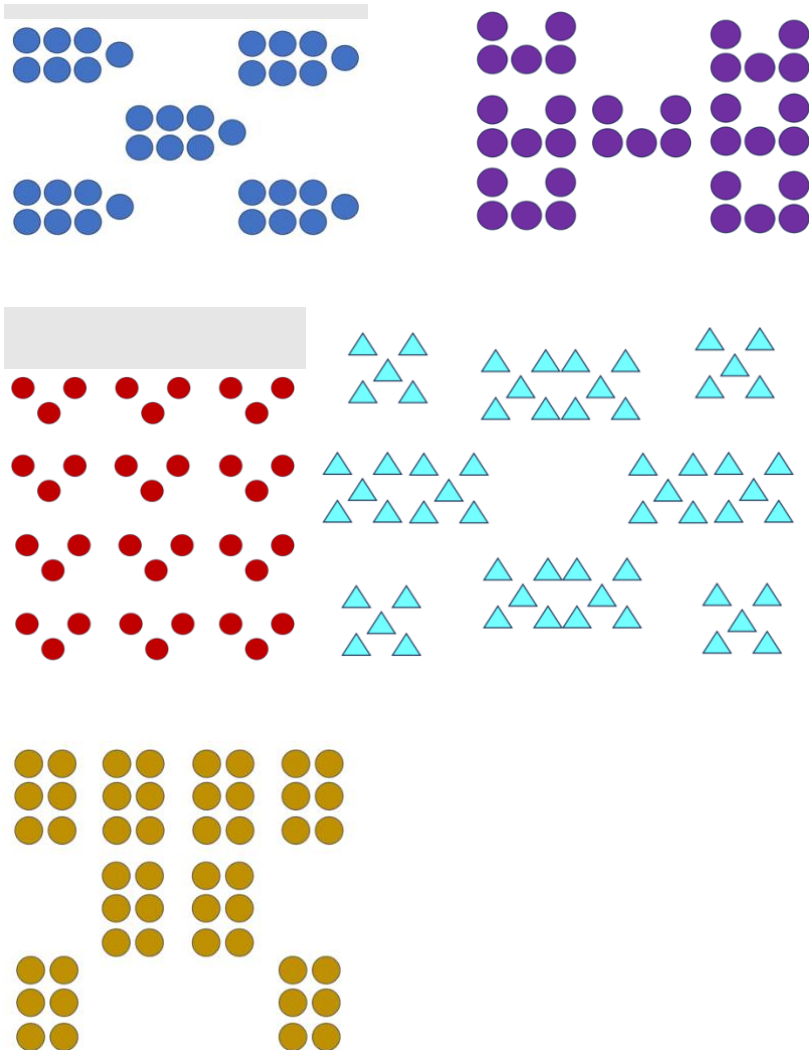
Big Ideas	Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
Curriculum Links	<ul style="list-style-type: none"> Recall multiplication facts to 10×10 and corresponding division facts Use the distributive, commutative, and associative properties
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Notice and use groupings to find a total Recall and apply multiplication facts Explain how they see an image
Mathematical language	Multiplication, groups of, commutative property, associative property, distributive property, equal to.
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> Explain to the students you are going to show them an image and they need to think about what they see, and how they see it. Show the image to the students for 3 seconds. Allow students time to visualize what they saw. Show the image again for another 3 seconds. Give more time for individual thinking. Ask students to turn and talk about how many dots they see and why. Display image again, keeping it displayed this time. Call on different students to share their thoughts. Record the different ways students saw the image. E.g., <div style="text-align: center;">  <p>3 groups of 4, 2 groups of 4, 3 groups of 4</p> </div>



9 groups of 4, but the middle group is missing.

7. Celebrate the different ways students notice this image.
8. You may wish to explicitly highlight one of the number properties students used. E.g., distributive property $(3 \times 4) + (2 \times 4) + (3 \times 4) = 8 \times 4$. Or make links between repeated addition and multiplication $(4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 8 \times 4)$

Other examples



PLACE-VALUE TO 100,000,000

876, 452, 019

What is this number? How do you know?

Big ideas	The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.
Curriculum links	<ul style="list-style-type: none"> Recognise, read, write, order, partition, recombine, and represent whole numbers up to 1,000,000
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Read numbers up to 1,000,000 Explain the place-value of each digit in numbers to 1,000,000
Mathematical language	Place-value, ones, tens, hundreds, thousands, ten-thousands, hundred-thousands, million, digit,
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> Display the image/ or write a number on a blank place-value house. Ask students to turn and tell a partner what the number is and why. Read the number together. Ensure students are using the correct language “eight hundred & seventy six million, four hundred and fifty two thousand and nineteen. Ask a series of questions that focus on the place-value of the numbers. E.g., “What does the 7 represent?”, “What is the value of the tens place?”, “How many ten-thousands are there?”, “What digit is in the hundred-thousand place? What is that digit's place-value?” Ask students to write the expanded form: $800,000,000 + 70,000,000 + 6,000,000 + 400,000 + 50,000 + 2,000 + 10 + 9$.
Other examples	<p>Repeat with numbers that you notice students need support with. Use numbers such as 804,400,072.</p> <p>Repeat with students writing a number for their partner, asking them to read it, then ask questions about the value of different places.</p>

READING AND EXPLAINING NUMBERS TO 1,000,000,000

954,507,893

What is this number?

How can you write and explain this in different ways?

Big ideas	The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.
Curriculum links	<ul style="list-style-type: none"> Recognise, read, write, order, partition, recombine, and represent whole numbers up to 1,000,000
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Read numbers up to 1,000,000,000 Explain the place-value of each digit in numbers to 1,000,000
Mathematical language	Place-value, ones, tens, hundreds, thousands, ten-thousands, hundred-thousands, million, digit,
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> Ask students, what is this number? Support the students to read the number correctly. Ask “how could you write or represent this number in different ways?” Give time for students to work with a partner to record ideas. Discuss and share the different ideas. Support students to discuss thousands, hundreds, tens, ones and make links to place, face, and total value. Link to the place value house as a representation and have this on the wall or whiteboard for students to refer to. Notice use of place value and the ability to see hundreds as ten tens and tens as ten ones. Draw connections to represent these within place value houses. To extend the task ask students questions like: “what would the number be if we changed the digit in the tens place to a 5?”, “what would the number be if we add 1000?”, “what would the number be if we moved each digit one place-value to the left/ or right?”
Other examples	<p>800,789,123 57,562 1,000,004 306,060</p>

CHORAL COUNTING

0.06	0.08	0.10	0.12	0.14
0.16	0.18	0.20	0.22	0.24
0.26	0.28	0.30		
		?		

Count by 0.02 starting at 0.06.

Big Ideas

Skip counting on the number line generates number patterns. Known elements in a pattern can be used to predict other elements.

Curriculum Links

- Recognise, read, write, order, partition, recombine, and represent whole numbers up to 1,000,000
- Add and subtract whole numbers and decimals to two places
- Use a rule to make predictions

Learning Outcomes

Students will be able to:

- Recognise patterns
- Add fractions with the same denominators
- Group ideas in a multiplicative ways
- Identify equivalent fractions or mixed numbers

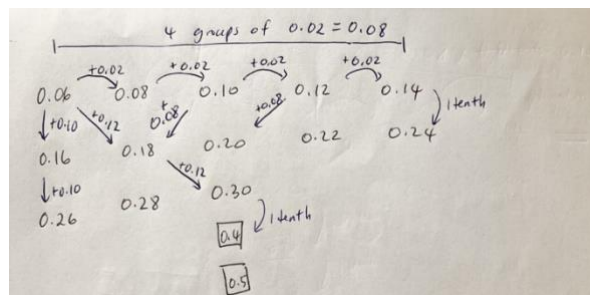
Mathematical language

Column, row, add, rule, position, pattern, more, less,

Teacher Notes

Instructions:

- Choose a count that you know students will be successful in, perhaps beginning with a fraction number and then later use decimal number.
- Think about how you will start the count.
- Give students a moment to think about the sequence prior to counting out loud.
- As the students count write down the count on the whiteboard. Pausing to support corrections when needed.



- After writing on the board, remember to start

discussions with the open-ended question, “What do you notice?”

- Give yourself space to listen to and can record students’ noticing’s.

- Look for an opportunity to take up one student idea and ask the whole class, “Why does that work?” or “How do you know?”
- Use talk moves (turn and talk, adding to someone’s idea, etc) to engage students with one another’s noticing’s.

Other examples

Ways to extend the 0.02 count over a series of days

- Make predictions about further points in the count

→ pattern grows by 0.02 or 2 hundredths or $\frac{2}{100}$
 ↘ pattern grows by 0.10 or 1 tenth or $\frac{1}{10}$
 ↘ pattern grows by 0.12 or 12 hundredths or $\frac{12}{100}$
 ↘ pattern grows by 0.08 or 8 hundredths or $\frac{8}{100}$
 ∴ all these numbers (0.02, 0.10, 0.12, 0.08) are multiples of 0.02

P	1	2	3	4	5	6	7	8
	0.06	0.08	0.10	0.12	0.14			

$0.02 \times \text{position number} + 0.04$
 $0.02P + 0.04$
 To find 78th number → $0.02(78) + 0.04$
 → $1.56 + 0.04 = 1.6$

Other counts:

Count in 3 tenths or $\frac{3}{10}$

4 groups of $\frac{3}{10} = \frac{12}{10}$, so $\frac{12}{10} + \frac{3}{10} = \frac{15}{10}$

$\frac{3}{10} + \frac{3}{10} = \frac{6}{10}$
 $\frac{6}{10} + \frac{3}{10} = \frac{9}{10}$
 $\frac{9}{10} + \frac{3}{10} = \frac{12}{10}$
 $\frac{12}{10} + \frac{3}{10} = \frac{15}{10}$
 $\frac{15}{10} + \frac{3}{10} = \frac{18}{10}$
 $\frac{18}{10} + \frac{3}{10} = \frac{21}{10}$
 $\frac{21}{10} + \frac{3}{10} = \frac{24}{10}$
 $\frac{24}{10} + \frac{3}{10} = \frac{27}{10}$
 $\frac{27}{10} + \frac{3}{10} = \frac{30}{10}$
 $\frac{30}{10} + \frac{3}{10} = \frac{33}{10}$
 $\frac{33}{10} + \frac{3}{10} = \frac{36}{10}$
 $\frac{36}{10} + \frac{3}{10} = \frac{39}{10}$
 $\frac{39}{10} + \frac{3}{10} = \frac{42}{10}$
 $\frac{42}{10} + \frac{3}{10} = \frac{45}{10}$
 $\frac{45}{10} + \frac{3}{10} = \frac{48}{10}$
 $\frac{48}{10} + \frac{3}{10} = \frac{51}{10}$
 $\frac{51}{10} + \frac{3}{10} = \frac{54}{10}$
 $\frac{54}{10} + \frac{3}{10} = \frac{57}{10}$
 $\frac{57}{10} + \frac{3}{10} = \frac{60}{10}$
 $\frac{60}{10} + \frac{3}{10} = \frac{63}{10}$
 $\frac{63}{10} + \frac{3}{10} = \frac{66}{10}$
 $\frac{66}{10} + \frac{3}{10} = \frac{69}{10}$
 $\frac{69}{10} + \frac{3}{10} = \frac{72}{10}$
 $\frac{72}{10} + \frac{3}{10} = \frac{75}{10}$
 $\frac{75}{10} + \frac{3}{10} = \frac{78}{10}$
 $\frac{78}{10} + \frac{3}{10} = \frac{81}{10}$
 $\frac{81}{10} + \frac{3}{10} = \frac{84}{10}$
 $\frac{84}{10} + \frac{3}{10} = \frac{87}{10}$

∴ ? = $\frac{87}{10}$ OR $8\frac{7}{10}$

Remember to plan your count before you begin and anticipate the patterns you think students will notice. Know what mathematical understandings could be reinforced in each count.

HOW MANY 1'S, 10'S, 100'S, 1000'S

How many?
 10s in 487,905?
 1s in 487,905
 1,000s in 487,905
 100s in in 487,905
 10,000 in 487,905
 100,000 in 487,905

Big ideas	The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.
Curriculum links	<ul style="list-style-type: none"> • In our number system, each place value is a power of 10, and this continues infinitely. • Recognise, read, write, order, partition, recombine, and represent whole numbers to 1,000,000
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Explain the number of 1's, 10's, 100's, 1000's, 10,000's, 100,000s in whole numbers.
Mathematical Language	Place value, base ten, ones, tens, hundreds, thousands, ten thousands, hundred thousands, millions, multiple, x10
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Present the first question "How many 10's in 28,107?" 2. Encourage students to turn and talk about what they think and why. Discuss students' ideas. Use materials such as a place-value house or place-value blocks to support student's explanations and understandings. 3. Reinforce the idea that understanding how many tens are in a number is different to reading the digit in the tens place (place naming). E.g., some students might say there are 0 tens in 28,107 because there is a 0 in the tens place. They may not realise there are 2,810 groups of ten in 28,107 4. Repeat for the other 4 questions. How many 1's etc. 5. Ensure students understand the place is 10 times bigger than the previous place when we move to the left. E.g. the hundreds place is ten times bigger than the tens place.
Other examples	<p>Complete this activity multiple times with a variety of different numbers.</p> <p>98,763,201</p> <p>3,481,190</p>

ORDER NUMBERS TO 100,000,000

87,299,999 12,024,160 299,999.0 659,818 3,204,160
 Order these numbers from biggest to smallest.

Big Ideas	Numbers, expressions, and measures can be compared by their relative values. Numerical and algebraic expressions can be compared using greater than, less than, or equal.												
Curriculum Links	<ul style="list-style-type: none"> Recognise, read, write, order, partition, recombine, and represent whole numbers up to 1,000,000 												
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Order whole numbers up to 1,000,000 Compare numbers using place-value 												
Mathematical language	Ones, tens, hundreds, thousands, tens of thousands, hundreds of thousands, add, subtract, place value, face value, total value, digit												
Teacher Notes	<p>Students need multiple opportunities to notice and generalise patterns within the structure of our number system.</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> Ask the students; What are these numbers? Support the students to read the number correctly. How can you order these numbers? Give students an opportunity to work in pairs and record and represent their reasoning. Allow students opportunities to discuss how the numbers are greater than or less than the others. Explore concepts, of place, face, and total value. Reinforce that the digit 0 can be used as a place holder. E.g., some students may have the misconception that 299,999.0 is larger than 299,999 because it looks longer, not realising that .0 represents there are no tenths. 												
Other examples	<p>Use combinations of different numbers.</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 33%;">45,876</td> <td style="width: 33%;">9,999</td> <td style="width: 33%;">460,000</td> </tr> <tr> <td>35,999</td> <td>999</td> <td>640,000</td> </tr> <tr> <td>26,010</td> <td>99,999</td> <td>604,000</td> </tr> <tr> <td>35,998</td> <td>99,909</td> <td>406,000</td> </tr> </table>	45,876	9,999	460,000	35,999	999	640,000	26,010	99,999	604,000	35,998	99,909	406,000
45,876	9,999	460,000											
35,999	999	640,000											
26,010	99,999	604,000											
35,998	99,909	406,000											

ROUND TO THE NEAREST...

Round to the nearest...

	whole number	...tenth	...hundredth
76.9875			
126.897			
98,290.011			

How do you know?

Big ideas	Decimals are a set of fractions that have powers of 10 as their denominators (e.g., $\frac{17}{10}$ or $\frac{107}{10}$) and that can be written as numbers using a decimal point (e.g., 0.7 or 0.07). A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line																
Curriculum links	<ul style="list-style-type: none"> ● On a number line, fractions and decimals occur between integers, and negative numbers are to the left of 0. ● recognise, read, write, represent, compare, and order fractions, decimals (to three places). 																
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Round decimals to the nearest whole number, tenth or hundredth. ● Justify their reasoning. 																
Mathematical language	Decimals, whole number, place value, tenths.																
Teacher Notes	<ul style="list-style-type: none"> ● Give an opportunity to discuss and justify with someone else before they share their ideas. ● Have a place value house for whole and decimal numbers on the wall or give to students to use if needed. 																
Other examples	<p>Here are some other examples you can use on other days you can explore one of these numbers or get them to try one of the three to justify.</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tbody> <tr> <td style="width: 15%;">Day 2</td> <td style="width: 20%;">3.231</td> <td style="width: 20%;">93.149</td> <td style="width: 20%;">33.645</td> </tr> <tr> <td>Day 3</td> <td>560.297</td> <td>5610.999</td> <td>301.732</td> </tr> <tr> <td>Day 4</td> <td>1299.777</td> <td>2003.182</td> <td>2110.618</td> </tr> <tr> <td>Day 5</td> <td>2999.847</td> <td>7165.487</td> <td>4999.956</td> </tr> </tbody> </table>	Day 2	3.231	93.149	33.645	Day 3	560.297	5610.999	301.732	Day 4	1299.777	2003.182	2110.618	Day 5	2999.847	7165.487	4999.956
Day 2	3.231	93.149	33.645														
Day 3	560.297	5610.999	301.732														
Day 4	1299.777	2003.182	2110.618														
Day 5	2999.847	7165.487	4999.956														

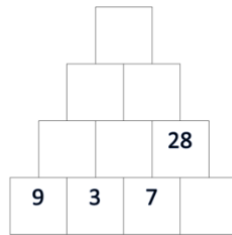
NUMBER LINE

The arrow is pointing at 500.
About where is 427? 540? 590?



Big Ideas	The set of real numbers is infinite, and each real number can be associated with a unique point on the number line.
Curriculum Links	<ul style="list-style-type: none"> ● Recognise, read, write, order, partition, recombine, and represent whole numbers to 1,000,000. ● Use representations to find, compare, explore, simplify, illustrate, prove, and justify patterns and variations
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Estimate the position of a number on a number line ● Use benchmarks to compare the size of a number
Mathematical language	More than, less than, between, approximate, ones, tens, hundred
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> ● Show the number line and explain that the arrow is pointing to 500. ● Ask students to discuss where the numbers are one at a time. ● Facilitate a discussion that draws on students explaining and justifying their reasoning. ● The teacher can annotate and record some benchmark numbers on the number line as the students explain their reasoning and refer to other positions on the number line. ● Engage students to debate about whether they agree or disagree with shared reasoning.
Other examples	<div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;"> <p>The arrow is pointing at 20. About where is 10? 22? 45?</p> <p>A horizontal number line with a vertical tick mark at the left end labeled '0'. An arrow points upwards from the number line to the value 20.</p> </div> <div style="text-align: center;"> <p>The arrow is pointing at 7×5. What are the endpoints?</p> <p>A horizontal number line with vertical tick marks at both ends. An arrow points upwards from the number line to the value 35.</p> </div> </div> <p>Repeat with a range of number lines, including fractions.</p>

MULTIPLICATION PYRAMID



Big Ideas	Multiplying by x is the inverse of dividing by x . Basic facts and algorithms for operations with rational numbers use notions of equivalence to transform calculations into simpler ones.
Curriculum Links	<ul style="list-style-type: none"> Recall multiplication facts to 10×10 and corresponding division facts Multiply two- and three- digit numbers Use the distributive, commutative and associative properties
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Recall basic multiplication and division facts to 10 Use the distributive property to solve 2-digit multiplication problems
Mathematical language	Multiplication, multiply, groups of, factor, product, equals, equivalent, distributive property, commutative property, associative property
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> In pairs, give sufficient time for students to fill in the blank squares. Have access to paper/whiteboard/pen to record representations & times table charts. Expect students to explain and justify as the teacher facilitates discussion to complete the pyramid on the board. Record all student solutions as they are shared as a representation alongside the pyramid. Make links between the inverse relationship between division and multiplication ($7 \times ? = 28$, $28 \div 7 = ?$). Connect to number properties. E.g., distributive <div style="text-align: center;"> <p>$(20 \times 20) + (8 \times 20) + (1 \times 20) + (1 \times 8)$</p> </div>
Other examples	

SQUARE NUMBERS

Think of a number.
 Square it.
 Subtract your starting number.
 Is the number you are left with an even or odd number?
 What do you notice and why?

Big ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways. There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.
Curriculum links	<ul style="list-style-type: none"> • Use a range of multiplicative and division strategies. • Solve equivalence problems. • Use inverse relationships and understanding of properties to solve problems. • Represent a variety of ways to explain how to achieve an answer. • Use prime numbers, common factors and multiples, and powers (including square roots)
Learning Outcomes: Students will be able to:	<ul style="list-style-type: none"> • Identify and describe the properties of prime, composite, square, and cube numbers and the divisibility rules for 2, 3, 5, 9, and 10. • Use words and symbols to describe and represent the properties of operations (commutative, distributive, associative, inverse, and identity)
Mathematical language	Number sentences, inverse property, division and multiplication. Commutative, associative, distributive, and identity properties work the same for all numbers.
Teacher Notes	<ul style="list-style-type: none"> • Square numbers are integers that can be expressed as the product of a whole number multiplied by itself. In other words, they are the result of squaring a whole number. • For example, 4, 9, and 25 are square numbers because they can be written as 2×2, 3×3, and 5×5, respectively. • Square numbers are called square numbers (or squared numbers) because they form the area of a square. The sides are an equal number of units in length so they make a square. • Teaching for conceptual understanding requires children to understand how and why a concept works, rather than learning abstract rules. • Explaining to children that square numbers are numbers multiplied by themselves is the final generalisation.

AREA MODEL

What equation is shown in this representation? Explain how you know.

	300	60	8
200	60,000	12,000	1,600
50	15,000	3,000	400
6	1,800	360	48

Can you show 337×568 or 142×13 using the area model.

Big Ideas	There are patterns in the products for multiplication facts																
Curriculum Links	<ul style="list-style-type: none"> Recall multiplication facts to 10×10 Multiply two- and three- digit numbers Use the distributive, commutative, and associative properties 																
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Recall basic multiplication and division facts to 10 Partition numbers into hundreds, tens and ones Use the distributive property to solve multiplication problems 																
Mathematical language	Area model, factors, multiplication, distributive property,																
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Display the image and give students time to turn and talk. 2. Share and discuss their ideas. 3. Reinforce that in this example students have used place-value partitioning to distribute the numbers into more manageable ones (e.g. $368 = 300 + 60 + 8$). 4. You may also need to recap how we can use known basic facts such as $6 \times 6 = 36$ to solve $60 \times 6 = 360$ 5. Record that this model represents application of the distributive property $(300 \times 200) + (300 \times 50) + (300 \times 6) +$ (Give students time to use the area model to represent other equations such as 33×56 or 142×13). 																
Other examples	<p style="text-align: center;">What equation does this represent?</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 15%;"></td> <td style="width: 35%;">500</td> <td style="width: 15%;">20</td> <td style="width: 35%;">83</td> </tr> <tr> <td style="border-top: 1px solid black;">300</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="border-top: 1px solid black;">90</td> <td></td> <td></td> <td></td> </tr> <tr> <td style="border-top: 1px solid black;">1</td> <td></td> <td></td> <td></td> </tr> </table>		500	20	83	300				90				1			
	500	20	83														
300																	
90																	
1																	

DIVISIBILITY

Using the Hundreds board / chart identifies the multiples of ...

What do you notice?
What patterns do you see?

Big ideas	Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways.
Curriculum links	Identify and describe the properties of prime, composite, and square numbers and the divisibility rules for 2, 3, 5, 9, and 10
Learning Outcomes: Students will be able to:	<ul style="list-style-type: none"> • Identify the divisibility rules of ... • Identify patterns in the multiples of ... • Use patterns in multiplication / timetables
Mathematical language	Divisible, digits, even, multiples
Teacher Notes	<ul style="list-style-type: none"> • Use a Hundred Board/Chart. • Ask the students in groups to find and circle the multiples of... Then discuss what they notice about those numbers. • Expect and encourage students to justify and explain their thinking to the group. • Highlight the importance of knowing these rules and patterns as they will support knowing basic facts and will allow students to estimate accurately when multiplying or dividing bigger numbers. • Encourage students to work out the divisibility rules for themselves. <p><u>Divisibility rules.</u></p> <ul style="list-style-type: none"> • All numbers are divisible by 1. • Divisible by 2, any arrangement of 4, 6, and 8 will be an even number. • Divisible by 3, the sum of the digits must be divisible by 3. • Divisible by 4, the number formed by the last two digits must be divisible by 4. • divisible by 5, the number must end in 5 or 0. • Divisible by 6, the number must be even and divisible by 3. • Divisible by 7, division rules apply • Divisible by 8, last three digits in a number divisible by 8. • Divisible by 9, the sum of the digits must be divisible by 9. • Multiples of even numbers are always even.
Other examples	Use across all of the basic facts numbers 1-12, then explore: <ul style="list-style-type: none"> - Divisibility / Multiples of 2 & 4 – Patterns and Generalisations - Divisibility / Multiples of 3,6, & 9 – Patterns and Generalisations.

DIVISION STRINGS – PARTIAL QUOTIENTS

<u>String 1</u>	<u>String 2</u>	<u>String 3</u>
$30 \div 3$	$200 \div 2$	$360 \div 4$
$24 \div 3$	$70 \div 2$	$24 \div 4$
$54 \div 3$	$270 \div 2$	$384 \div 4$
Big Ideas	For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra. Division algorithms use numerical estimation and the relationship between division and multiplication to find quotients	
Curriculum Links	<ul style="list-style-type: none"> ● Multiplication and division can involve equals groups, rates, comparisons, combinations, part-whole relationships, areas and volumes ● Recall multiplication facts to 10 x 10 and corresponding division facts ● Divide whole numbers by one- or two- digit divisors ● Find factors of numbers up to 100 	
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Break numbers into partial dividends ● Find multiples and factors ● Apply multiplication facts to division problems 	
Mathematical language	divide, division, divisor, dividend, quotient, inverse, multiplication, multiply, groups of, factor, product, equivalent, distributive property	
Teacher Notes	<p>These number strings support students to solve problems by breaking the dividend into smaller partial dividends (distributive property). These partial dividends must be divisible by the divisor.</p> <p><i>Dividend:</i> the number that will be divided <i>Divisor:</i> the number the dividend is being divided by <i>Quotient:</i> product/ answer</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Display the first equation. E.g., $30 \div 3$. Encourage students to turn and talk about the quotient and to justify their reasoning. 2. Students may draw on known multiplication/division facts or need access to a basic facts chart. 3. Display the second equation. E.g., $24 \div 3$. Encourage students to turn and talk about the quotient and to justify their reasoning. 4. Display the final equation. E.g., $54 \div 3$. Encourage students to look for relationships between the previous equations in the string and to use them to solve the next equation. 5. Record all student solutions as they are shared as a representation on the board alongside the number string. 6. Reinforce that $(30 \div 3) + (24 \div 3) = 54 \div 3$ 7. Ask students “why might it be useful to break up a large division equation into smaller ones?” or “why are 30 and 24 	

	<p>useful numbers to choose?” (because they are both factors of the divisor).</p> <p>8. Repeat with other strings.</p>
Other examples	<div style="background-color: #e0e0e0; padding: 10px; margin-bottom: 10px;"> $40 \div 4$ $30 \div 3$ $40 \div 4$ $16 \div 4$ $90 \div 3$ $24 \div 4$ $56 \div 4$ $93 \div 3$ $64 \div 4$ </div> <div style="background-color: #e0e0e0; padding: 10px; margin-bottom: 10px;"> $5 \div 5$ $160 \div 8$ $100 \div 4$ $30 \div 15$ $10 \div 5$ $16 \div 8$ $200 \div 4$ $90 \div 15$ $25 \div 5$ $400 \div 8$ $40 \div 4$ $300 \div 15$ $50 \div 5$ $80 \div 8$ $16 \div 4$ $150 \div 15$ $75 \div 5$ $496 \div 8$ $256 \div 4$ $540 \div 15$ </div> <div style="background-color: #e0e0e0; padding: 10px;"> $400 \div 4$ $130 \div 13$ $100 \div 20$ $80 \div 4$ $26 \div 13$ $200 \div 20$ $16 \div 4$ $52 \div 13$ $400 \div 20$ $496 \div 4$ $195 \div 13$ $500 \div 20$ </div>

MULTIPLICATION STRINGS

String 1

$$3 \times 4$$

$$30 \times 4$$

$$29 \times 4$$

String 2

$$5 \times 9$$

$$5 \times 90$$

$$5 \times 89$$

$$6 \times 89$$

Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Equations show relationships of equality between parts on either side of the equal sign.
Curriculum Links	<ul style="list-style-type: none"> ● Recall multiplication facts to 10 x 10 and corresponding division facts ● Multiply two- and three- digit numbers ● Use the distributive, commutative and associative properties
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Use known facts to solve multiplication problems • Identify relationships between equations
Mathematical language	Multiplication, groups of, factor, product, equals, equivalent, distributive property, commutative property, associative property,
Teacher Notes	<p>These multiplication strings have been designed to encourage students to use known facts and place-value to make solving larger problems easier. Provide access to timetables card if students require them.</p> <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Display the first equation (3×4) and give students time to turn and talk about the product and to justify their reasoning. • Expect students to explain and justify as the teacher facilitates discussion about solution strategies. • Display the second equation (30×4). Encourage students to look for a relationship between the previous equation in the string and to use this to solve the next equation. E.g., 30×4 is ten times bigger than 3×4. Record all student solutions as they are shared as a representation on the board alongside the number string. • Display the final equation (29×4). Ask students “how could you use 3×4 and 30×4 to solve 29×4?” • Name the number properties if they arise e.g. associative, commutative, distributive.

Other examples

$2 \times 9 =$

$20 \times 9 =$

$19 \times 9 =$

$6 \times 8 =$

$6 \times 80 =$

$6 \times 79 =$

$3 \times 11 =$

$30 \times 11 =$

$27 \times 11 =$

$3 \times 50 =$

$50 \times 50 =$

$53 \times 50 =$

$53 \times 49 =$

$2 \times 25 =$

$4 \times 25 =$

$8 \times 25 =$

$10 \times 25 =$

$16 \times 25 =$

$2 \times 7 =$

$4 \times 7 =$

$40 \times 7 =$

$38 \times 7 =$

$3 \times 10 =$

$3 \times 50 =$

$3 \times 100 =$

$3 \times 149 =$

$5 \times 200 =$

$20 \times 200 =$

$25 \times 200 =$

$25 \times 199 =$

$6 \times 20 =$

$6 \times 100 =$

$6 \times 120 =$

$6 \times 119 =$

MULTIPLICATION STRINGS – PLACE VALUE

	$4 \times 6 =$	
$40 \times 6 =$	$4 \times 60 =$	$40 \times 60 =$
$400 \times 6 =$	$4 \times 600 =$	$400 \times 600 =$
$4000 \times 6 =$	$4 \times 6000 =$	$4000 \times 6000 =$

Big Ideas	<p>There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties.</p> <p>Equations show relationships of equality between parts on either side of the equal sign.</p>																								
Curriculum Links	<ul style="list-style-type: none"> ● Recall multiplication facts to 10 x 10 and corresponding division facts ● Multiply two- and three- digit numbers ● Use the distributive, commutative and associative properties 																								
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Use known facts to solve multiplication problems • Identify relationships between equations 																								
Mathematical language	Multiplication, groups of, factor, product, equals, equivalent, distributive property, commutative property, associative property,																								
Teacher Notes	<p>These multiplication strings have been designed to encourage students to use known facts and place-value to make solving larger problems easier. Provide access to timetables card if students require them.</p> <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Display the first equation (4×6) and give students time to turn and talk about the product and to justify their reasoning. • Expect students to explain and justify as the teacher facilitates discussion about solution strategies. • Display the second equation (40×6). Encourage students to look for a relationship between the previous equation in the string and to use this to solve the next equation. E.g., 40×6 is ten times bigger than 4×6. Record all student solutions as they are shared as a representation on the board alongside the number string. • Ask the students what they notice with the answers as more of the string is complete. • Explicitly discuss that if students can access basic facts they can solve challenging multiplication tasks. 																								
Other examples	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 33%;"></td> <td style="width: 33%; text-align: center;">$7 \times 9 =$</td> <td style="width: 33%;"></td> </tr> <tr> <td style="text-align: center;">$70 \times 9 =$</td> <td style="text-align: center;">$7 \times 90 =$</td> <td style="text-align: center;">$70 \times 90 =$</td> </tr> <tr> <td style="text-align: center;">$700 \times 9 =$</td> <td style="text-align: center;">$7 \times 900 =$</td> <td style="text-align: center;">$700 \times 900 =$</td> </tr> <tr> <td style="text-align: center;">$7000 \times 9 =$</td> <td style="text-align: center;">$7 \times 9000 =$</td> <td style="text-align: center;">$7000 \times 9000 =$</td> </tr> <tr> <td style="width: 33%;"></td> <td style="width: 33%; text-align: center;">$12 \times 11 =$</td> <td style="width: 33%;"></td> </tr> <tr> <td style="text-align: center;">$120 \times 11 =$</td> <td style="text-align: center;">$12 \times 110 =$</td> <td style="text-align: center;">$120 \times 110 =$</td> </tr> <tr> <td style="text-align: center;">$1200 \times 11 =$</td> <td style="text-align: center;">$12 \times 1100 =$</td> <td style="text-align: center;">$1200 \times 1100 =$</td> </tr> <tr> <td style="text-align: center;">$12000 \times 11 =$</td> <td style="text-align: center;">$12 \times 11000 =$</td> <td style="text-align: center;">$12000 \times 11000 =$</td> </tr> </table>		$7 \times 9 =$		$70 \times 9 =$	$7 \times 90 =$	$70 \times 90 =$	$700 \times 9 =$	$7 \times 900 =$	$700 \times 900 =$	$7000 \times 9 =$	$7 \times 9000 =$	$7000 \times 9000 =$		$12 \times 11 =$		$120 \times 11 =$	$12 \times 110 =$	$120 \times 110 =$	$1200 \times 11 =$	$12 \times 1100 =$	$1200 \times 1100 =$	$12000 \times 11 =$	$12 \times 11000 =$	$12000 \times 11000 =$
	$7 \times 9 =$																								
$70 \times 9 =$	$7 \times 90 =$	$70 \times 90 =$																							
$700 \times 9 =$	$7 \times 900 =$	$700 \times 900 =$																							
$7000 \times 9 =$	$7 \times 9000 =$	$7000 \times 9000 =$																							
	$12 \times 11 =$																								
$120 \times 11 =$	$12 \times 110 =$	$120 \times 110 =$																							
$1200 \times 11 =$	$12 \times 1100 =$	$1200 \times 1100 =$																							
$12000 \times 11 =$	$12 \times 11000 =$	$12000 \times 11000 =$																							

TRUE OR FALSE (POWERS)

$$8 \times 8 \times 8 \times 8 = 8^4$$

Explain and justify why.

Big ideas	Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways.
Curriculum links	<ul style="list-style-type: none"> ● Use a range of multiplicative strategies when operating on whole numbers. ● Generalise properties of multiplication and division with whole numbers. ● Use prime numbers, common factors and multiples, and powers (including square roots).
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Identify that a power is represented by a base number and an exponent and that a power is the product of multiplying a number by itself. ● Calculate powers of numbers. ● Explain and justify patterns and relationships in powers of numbers.
Mathematical language	Power, base number, exponent, product, digit, conjecture, tens, hundreds, thousands.
Teacher Notes	<u>Instructions:</u> <ul style="list-style-type: none"> ● Display the question for all students to see. ● Allow students time to turn and talk and identify the patterns that they notice. ● Discuss student's ideas and scribe their thinking on the board for them to see. ● Encourage the students to solve the problem. What was this look like? ● Ask the students – when would you use powers in real life?
Other examples	$(4 \times 4 \times 4 \times 4) + (4 \times 4 \times 4 \times 4 \times 4) = 4^3 + 4^5$ $(8 \times 8) + (9 \times 9 \times 9 \times 9) + (2 \times 2 \times 2) = 8^2 + 9^4 + 2^3$ $(5 \times 5 \times 5 \times 5) + (7 \times 7 \times 7 \times 7) = 12^8$

TRUE OR FALSE (ADDING POWERS)	
$(10 \times 10 \times 10) + (10 \times 10 \times 10 \times 10) = 10^3 + 10^4$ Explain and justify why.	
Big ideas	Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways.
Curriculum links	<ul style="list-style-type: none"> ● Use a range of multiplicative strategies when operating on whole numbers. ● Generalise properties of multiplication and division with whole numbers. ● Use prime numbers, common factors and multiples, and powers (including square roots).
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Identify that a power is represented by a base number and an exponent and that a power is the product of multiplying a number by itself. ● Calculate powers of numbers. ● Explain and justify patterns and relationships in powers of numbers.
Mathematical language	Power, base number, exponent, product, digit, conjecture, tens, hundreds, thousands.
Teacher Notes	<u>Instructions:</u> <ul style="list-style-type: none"> ● Display the question for all students to see. ● Allow students time to turn and talk and identify the patterns that they notice. ● Discuss student's ideas and scribe their thinking on the board for them to see. ● Encourage the students to solve the problem. What would this look like?
Other examples	$(4 \times 4 \times 4 \times 4) + (4 \times 4 \times 4 \times 4 \times 4) = 4^3 + 4^5$ $(8 \times 8) + (9 \times 9 \times 9 \times 9) + (2 \times 2 \times 2) = 8^2 + 9^4 + 2^3$ $(5 \times 5 \times 5 \times 5) + (7 \times 7 \times 7 \times 7) = 12^8$

IF... THEN...

<p>If... $60 \times 30 = 180$ and $30 \times 60 = 180$ Then... $180 \div 60 = 30$ and $180 \div 30 = 60$ Could this pattern work for any multiplication sentence? Explore.</p>	
Big ideas	Division facts can be found by thinking about the related multiplication fact.
Curriculum links	<ul style="list-style-type: none"> Identify and describe the properties of prime, composite, square, and cube numbers and the divisibility rules for 2, 3, 5, 9, and 10. Use words and symbols to describe and represent the properties of operations (commutative, distributive, associative, inverse, and identity).
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> Recognise and explore patterns, and make conjectures and draw conclusions about them. Identify relationships, including similarities, differences, and new connections. Look for patterns and regularities that can be applied in another situation or are always true. Make and test conjectures, using reasoning and counter examples to decide if they are true or not.
Mathematical language	Multiplication, division, inverse, related facts, commutative property
Teacher Notes	<p>Every multiplication sentence has two related division sentences ($a \times b = c$ so $c \div b = a$ and $c \div a = b$).</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> Ask students what they notice about the numbers in the equations. Expect students to justify and reason giving explanations. Highlight student thinking that draws on the inverse relationship. Give students time to form the own set of related multiplication and division sentences. Share these with the class. Encourage students to realise that if they know their multiplication facts they can easily solve division by using the inverse. To extend the task discuss the generalization of $a \times b = c$ so $c \div b = a$.
Other examples	<p>If... $4 \times 9 = 36$ and $\underline{\quad} \times \underline{\quad} = \underline{\quad}$ Then... $36 \div 4 = 9$ and $36 \div 9 = 4$</p> <p>If... $15 \times 10 = 150$ and $10 \times 15 = 150$ Then... $150 \div 15 = 10$ and $\underline{\quad} \div \underline{\quad} = 15$</p> <p>If... $25 \times \underline{\quad} = 2500$ and $\underline{\quad} \times 25 = 2500$ Then... $2500 \div 25 = 100$ and $\underline{\quad} \div \underline{\quad} = \underline{\quad}$</p>

WHAT'S THE POINT? POSITIVE AND NEGATIVE INTEGERS



What number is each arrow pointing to on the number line?
Justify your thinking

Big ideas	Mathematical situations can be represented as equations which include both positive and negative integers. A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.
Curriculum links	<ul style="list-style-type: none"> • Order and compare integers on a number line
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ≠ Use a number line to represent the relationship between positive and negative integers in equations ≠ Explain and justify the role of zero as neither positive nor negative ≠ Explain and justify the use of – as an operation symbol (subtraction) and direction symbol (direction, size of movement) for negative numbers
Mathematical language	Integers, negative number, positive number
Teacher Notes	<ul style="list-style-type: none"> • Purpose: to identify integers on a number line <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Show students one number line at a time. Emphasise the need to justify their reasoning. The red arrow is because.... • Notice what benchmarks students are using. E.g. zero would be here, therefore I think the red arrow is pointing to.... • Encourage students to talk with a buddy. Pay attention to any mathematical argumentation and highlight this with the larger group. Who thinks the same, who thinks differently and why? Who has changed their thinking and why? What convinced you?
Other examples	<div style="text-align: center;"> <p>The first number line has tick marks at -37 and -13. It has three arrows: a green arrow pointing to a value between -37 and -13, a red arrow pointing to a value between -13 and -37, and a blue arrow pointing to a value between -13 and -37.</p> <p>The second number line has tick marks at -25 and 25. It has three arrows: a green arrow pointing to a value between -25 and 25, a blue arrow pointing to a value between 25 and -25, and a red arrow pointing to a value between -25 and 25.</p> </div> <p>This idea can be extended to include decimals. Challenge students to explore the idea of negative integers with decimals. Can you have -3.25? Why or why not?</p> <p>What about fractions? Can we have $-\frac{1}{2}$? Where does this sit on the number line?</p>

WHAT IS THE STORY? POSITIVE AND NEGATIVE INTEGERS

What could the story be? $-2 + 6 = 4$ $13 - 16 = -3$	
Big ideas	Mathematical situations can be represented as equations which include both positive and negative integers. A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.
Curriculum links	Add and subtract integers Make statements and justify how to add and subtract integers
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ∉ Solve simple addition and subtraction equations ∉ Use a number line to represent the relationship between positive and negative integers in equations ∉ Explain and justify the role of zero as neither positive nor negative ∉ Explain and justify the use of $-$ as an operation symbol (subtraction) and <u>direction symbol</u> (direction, size of movement) for negative numbers
Mathematical language	Integers, negative number, positive number, rise, fall, below
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> ● Start with one equation and students to share their ideas with their buddies. ● Encourage students to then show this equation on an empty number line using arrows and notation to represent the equation, to then discuss what the real life context could be. ● Notice the students that are using the correct language of negative numbers. ● Represent student explanations on the board when facilitating whole class discussion. ● If needed have a discussion of negative number brainstorm prior to starting: above and below sea level, weather reports, freezing details on food packages, money, going down a lift into a parking lot, golf scores etc.
Other Examples	Use a selection of both positive and negative integers in different combinations. $-5 + -5 = -10$ $-3 - 6 = -9$ $-4 - -7 = 3$ $4 - -4 = 8$ <p>Equations with missing variables e.g. $-3 + _ = 4$ or no solutions will increase the challenge to this task.</p>

FRACTION – NUMBER LINE



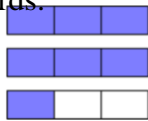
What fractions could go on this number line?

Big ideas	<p>A fraction describes the division of a whole into equal parts. The bottom number in a fraction tells how many equal parts the whole our unit is divided into. The top number tells how many equal parts are indicated.</p> <p>Each fraction can be associated with a unique point on the number line, but not all of the points between integers can be named by fractions.</p>
Curriculum links	<ul style="list-style-type: none"> On a number line, fractions and decimals occur between integers, and negative numbers are to the left of 0.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Notice fraction points on a number line Represent fractions in their simplest form Add and subtract fractions with related denominators Justify and explain their thinking
Mathematical Language	<p>Fraction numbers like one-twelfth, two-twelfths, six-twelfths and so on.</p> <p>Equivalent fractions, denominators, numerators</p>
Teacher Notes	<ul style="list-style-type: none"> Key question to ask students “What do you notice?” <u>Instructions:</u> <ol style="list-style-type: none"> Place the number line up on the board. Ask the students to turn and talk and discuss what numbers are between 0 and 1 on this number line. The teacher draws attention to the fact that twelve spaces are between zero and one; through questioning, the class concludes that each space has a length of one-twelfth of the whole. Through further questioning, the class determines that successive lines should be called <i>one-twelfth</i>, <i>two-twelfths</i>, <i>three-twelfths</i> and so on. Use talk moves to facilitate participation and develop understanding. Discuss what students notice when teacher asks students to crouch down and then calls out the fraction. What are the equivalent fractions? Which fraction is bigger and why?
Other examples	<p>Use a variety of different sized number lines and with different sized gaps between the numbers.</p> <p>Repeat this warm up using whole numbers, for example a number line between numbers 20 and 21.</p>

TRUE OR FALSE- CONVERT IMPROPER FRACTIONS

$$2\frac{1}{3} = \frac{7}{3}$$

Materials- Fraction tiles

Big ideas	<p>A fraction describes the division of a whole (region, set, segment) into equal parts.</p> <p>The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.</p> <p>A fraction is relative to the size of the whole or unit.</p> <p>A fraction describes division.($a/b = a \div b$, a & b are integers & $b \neq 0$), and it can be interpreted on the number line in two ways. For example, $2/3 = 2 \div 3$. On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $1/3$ of a unit ($2 \times 1/3$) or $1/3$ of 2 whole units ($1/3 \times 2$); each is associated with the same point on the number line. (Rational number)</p>
Curriculum links	<ul style="list-style-type: none"> • On a number line, fractions and decimals occur between integers, and negative numbers are to the left of 0. • Represent fractions in their simplest form.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Convert improper fraction to mixed fraction. • Explain and justify their thinking.
Mathematical Language	Whole, fraction, improper fraction, mixed fraction, denominator, numerator
Teacher Notes	<ol style="list-style-type: none"> 1. Ask students to name some proper and mixed fractions. Record it on the whiteboard. 2. Write the mixed fraction and improper fraction on the whiteboard and ask students to explain and justify if it's true or false. 3. When the students share back, record their justification including representation on the whiteboard. 4. Highlight to the students that an improper fraction has a numerator greater than the denominator. 5. Draw this representation on the whiteboard and count the thirds. <div style="text-align: center;">  </div> 6. Write the fraction as a mixed fraction by counting how many wholes and parts.
Other examples	<p>True or False</p> $\frac{17}{5} = 2\frac{7}{5} \quad \frac{11}{4} = 3$

FRACTIONS: CONVERTING IMPROPER FRACTIONS (NUMBER LINE)

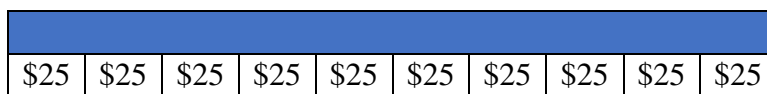
Where does it go? How do you know?
Convert the improper fraction into a mix number and place on the number line.



$\frac{22}{4}$

Big ideas	The world is full of patterns and structures that we use mathematics and statistics to understand.
Curriculum links	<ul style="list-style-type: none"> On a number line, fractions and decimals occur between integers, and negative numbers are to the left of 0.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> represent decimals, fractions, and percentages using both discrete and continuous models.
Mathematical Language	Equivalent fractions, denominator, numerator, number line, mixed number, improper fraction
Teacher Notes	<ul style="list-style-type: none"> Fractions number line is for students to recognise that the whole is the distance between the students representing zero and one. Key question to ask students “What do you notice?” and “How do you know?” <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Notice the students that are using fractional language in their explanations to share their thinking with the class. Attached are some examples below to solve the number line. <p>Note: students may use multiplication or division to solve as well.</p> <div style="text-align: center; margin: 10px 0;"> </div> <ul style="list-style-type: none"> Use talk moves to support participation and facilitate discussion.
Other examples	<p>Other improper fractions that will generate a discussion</p> $\frac{67}{8} \text{ or } \frac{17}{6} \text{ or } \frac{39}{15} \text{ or } \frac{12}{9}$ <p>Have two fractions that they need to convert into mixed numbers and order them on the number line. Which is bigger and how do you know?</p>

FIND THE WHOLE AMOUNT WHEN GIVEN A PERCENTAGE OR FRACTION AMOUNT.



70% of \$___ = \$175

Big ideas	<p>Different real-world interpretations can be associated with the product of a whole number and fraction (decimal), a fraction (decimal) and whole number, and a fraction and fraction (decimal and decimal).</p> <p>Different real-world interpretations can be associated with division calculations involving fractions (decimals).</p>
Curriculum links	<ul style="list-style-type: none"> • Recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages. • Represent fractions in their simplest form
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Explain and justify the comparison of a part to the whole. • Use representations to find the whole when a percentage and fraction is given.
Mathematical language	Whole number, fraction, fractional number, decimal number, rational number, equal, equivalent, percentage,
Teacher Notes	<ul style="list-style-type: none"> • Show the bar model to the students and ask them to turn and talk about what they notice about the model. Monitor the use of decimal and fractional language. • Ask students work with a buddy to find 70% of \$___ = \$175 • Have materials available or mini whiteboards or paper if students want to draw their representations. • Ask the students to explain and justify that the total of the bar will be 100% and \$250. • If students do not draw the bar model, colour 70% of the bar and add the \$25 to prove that 70% of the bar = \$175 and the value of the whole bar is \$250= 100% • Facilitate the students to notice that the value of the 30% is \$75. \$175 and \$75 = \$250 • Emphasise the idea that $70\% + 30\% = 100\%$ so therefore, $\frac{70}{100} + \frac{30}{100} = \frac{100}{100}$ or $\frac{7}{10} + \frac{3}{10} = \frac{10}{10}$
Other Examples	<p>Ask students to draw bar models or other representations to solve these equations:</p> <ol style="list-style-type: none"> 1. 90% of _____ = \$360 2. $\frac{6}{8}$ of _____ = \$48 3. 45% of _____ = \$1.90 4. $\frac{5}{7}$ of _____ = \$22.50 5. 60% of _____ = \$21

FRACTIONAL PART OF A SET

You have read 231 pages of your book. You celebrate reading one third of your book.

What fraction of your book do you have left to read?
How many pages is that?

How many pages in the whole book?

How many ways can you represent your thinking?

Big ideas	A fraction describes the division of a whole (region, set, segment) into equal parts.
Curriculum links	<ul style="list-style-type: none"> • Fractions show parts of a whole in a region, a measurement, or a set of objects. The same amount (e.g., a half or a quarter) can be shown by equivalent fractions. • Find a unit fraction of a whole (e.g., a region, measurement, or set of objects), and add unit fractions with the same denominator. • recognise, read, write, represent, and order halves, thirds, quarters, fifths, sixths, sevenths and eighths
Learning Outcomes: Students will be able to:	<ul style="list-style-type: none"> • Put two, four and eight equal parts (units) together to make one whole. • Count or add fractional parts to make one whole. • Combine and recombine different units of fractions to make one whole.
Mathematical language	whole, quarters, fourths, thirds, sevenths, equal, equivalent, fair share, partitioning, numerator, denominator
Teacher Notes	<p>The purpose of this activity is for students to find the fraction of the whole set. Student's practice explaining and justifying.</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Show the statement. 2. Give a short time for individual thinking, then ask students to explain their thinking to a buddy. 3. Encourage students to use a variety of representations eg. Materials, drawings, fraction tiles... 4. Encourage the students to recognise that $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3}$ 5. So $\frac{1}{3}$ is 231, $\frac{2}{3} = 462$ and the whole book is 693. Or $200 \times 3 = 600 + 30 \times 3 = 90 + 1 \times 3 = 3$
Other examples	<p>You have read 2102 pages of your book. You celebrate reading one quarter of your book.</p> <p>How many pages in the whole book?</p>

WOULD YOU RATHER? (FRACTIONS AND PERCENTAGES)

Would you rather have 24% of a pizza or 60% of half a pizza?

Discuss with your peers.

Big ideas	<p>Numbers can be described in many different ways including as fractions.</p> <p>The whole is important in naming fractions. A fraction is relative to the size of the whole or unit.</p> <p>A comparison of a part to the whole can be represented using a fraction.</p> <p>A fraction describes the division of a whole (region, set, segment) into equal parts.</p> <p>Each fraction can be associated with a unique point on a number line.</p> <p>There is no least or greatest fraction on the number line.</p> <p>There are an infinite number of fractions between any two fractions on the number line.</p>
Curriculum links	<ul style="list-style-type: none"> • Recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages
Learning Outcomes: Students will be able to:	<ul style="list-style-type: none"> • Order and compare fractions. • Find equivalent fractions.
Mathematical language	whole, thirds, twelfths, eighths, fifteenths, twentieths, fraction, equal, equivalent, greater than, less than, numerator, denominator
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Display the statement where all students can see. • Encourage students to discuss their thinking with their group. • If students want to represent their thinking, provide mini whiteboards or paper to allow them the opportunity. • Notice students that are able to justify their thinking using fractional language. • Select students to share back to facilitate discussion amongst the whole class, asking the question “Do you agree or disagree with their reasoning?” “Why?”. • To extend this warm up further into an independent task, ask the students to represent their thinking in multiple ways.
Other examples	<ul style="list-style-type: none"> • Would you rather the length of your overseas holiday be 2% of a year or $\frac{1}{3}$ of a month long? • Would you rather a third of a $\frac{1}{4}$ of a cake or 15% of the cake?

MIX AND MATCH : FRACTIONS, DECIMALS AND PERCENTAGES

Match the number in the first column with its equivalent value in the second column.

0.666666	$\frac{9}{20}$
$\frac{5}{10}$	one-quarter
45%	50%
60%	$\frac{16}{20}$
0.25	$\frac{2}{3}$
Eighty percent	0.6

Big ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.
 The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.
 Each fraction can be associated with a unique point on a number line.

Curriculum links

Recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages.

Learning Outcomes:
Students will be able to:

- Order and compare fractions.
- Find equivalent fractions.

Mathematical language

Whole, thirds, twelfths, eighths, fifteenths, twentieths, fraction, equal, equivalent, greater than, less than, numerator, denominator

Teacher Notes

Instructions:

- Give the students the time to discuss with their peers.
- Notice students who are able to explain their thinking using fractional language. Share their reasoning with the class.
- Ask students if they agree or disagree and why?
- If using as an independent activity expect the students to represent their thinking in multiple ways.

Other examples

$\frac{7}{10}$	0.1
2.6	$\frac{6}{10}$
0.15	$2\frac{6}{10}$
10%	0.25
$\frac{11}{44}$	70%
60%	15%

FRACTIONS, DECIMALS, PERCENTAGES

$$\frac{3}{5} = 60\% = 0.6$$

Is the number sentence true or false. Use representations to explain and justify.

(Materials: Empty number lines, fraction tiles)

Big ideas	<ul style="list-style-type: none"> Percent is relative to the size of the whole. A percent is a special type of ratio where a part is compared to a whole and the whole is 100. A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.
Curriculum links	<ul style="list-style-type: none"> On a number line, fractions and decimals occur between integers, and negative numbers are to the left of 0. Recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Explain and justify the comparison of a part to the whole Represent reasoning using different forms of notation, including words Use benchmark fractions to convert other fractions, percentages and decimals
Mathematical language	Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent
Teacher Notes	<p>This activity will support the students to convert between fractions, percentages and decimals.</p> <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Place the equation on the board or screen. Give students the opportunity to discuss and represent their thinking with their peers. When students share back, monitor for students using vocabulary within the language of rational number and that percent is out of one hundred. Draw the different representations used by students on the whiteboard explaining why 60 percent is the same as three fifths and 0.6. If students do not use a number line, show them the number line as another representation.
Other Examples	$\frac{8}{12} = 67\% = 0.67$ $\frac{9}{20} = 45\% = 0.45$ $\frac{18}{20} = 90\% = 0.9$

DECIMAL ADDITION – MISSING ADDENDS

$$\underline{\quad} + \underline{\quad} = 9.12024$$

What could the missing addends be in this sum?

Big Ideas	Decimals are a set of fractions that have powers of 10 as their denominators and that can be written as numbers using a decimal point. Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.
Curriculum Links	<ul style="list-style-type: none"> • Add and subtract decimal numbers to two places • Solve open number sentences and true or false number sentences involving equality or inequality
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Add tenths and hundredths • Solve open-ended addition problems • Justify and explain their thinking.
Mathematical language	Place value, base ten, tenths, hundredths, thousandths, decimal, equals, equivalent, addition, addend, sum
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Recap that an addend is a number that is added to another one. 2. Give students sufficient time to record some possible solutions. 3. Call on students to explain possible solutions and ensure correct place-value language is used. E.g., 3 wholes & 2 hundredths + 1 whole and 4 thousandths. 4. Push for students to provide reasoning and justification about why their two missing addends are equal to 4.024 (they may need access to materials to prove this). 5. Facilitate discussion by asking students if they agree or disagree with the reasoning shared. 6. Refer to the place value house throughout the discussion to make connections to the value of the digits and to highlight the base ten number system.
Other examples	$\underline{\quad} + \underline{\quad} = 12.632$ $6.71 = \underline{\quad} + \underline{\quad}$ $\underline{\quad} + \underline{\quad} = 7.985$ $10.10 = \underline{\quad} + \underline{\quad}$ $\underline{\quad} + \underline{\quad} = 0.406$ $\underline{\quad} + \underline{\quad} = 0.030$

MULTIPLY FRACTIONS AND DECIMALS BY WHOLE NUMBERS

	$36 \times 63 =$ $0.36 \times 6.3 =$ $36 \times 0.63 =$ $3.6 \times 63 =$ $0.36 \times 0.63 =$
Big ideas	The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers. Division with a decimal divisor is changed to an equivalent calculation with a whole number divisor by multiplying the divisor and dividend by an appropriate power of ten
Curriculum links	<ul style="list-style-type: none"> • Use multiplicative understanding of place value to solve multiplication and division problems with decimal numbers.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Represent reasoning using different forms of notation, including symbols and words. • Solve problems involving decimal numbers by multiplying and explain and justify the solution. • Represent reasoning to explain and justify place value involving decimal numbers
Mathematical language	Whole number, fraction, fractional number, decimal number, rational number, equal, equivalent
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Have all equations visible for all pairs or small groups to access. • Have materials available or mini whiteboards or paper if students want to draw their representations. • Ask the students to explain and justify what is happening to the value of each equation when multiplied. • If students do not use a number line, show them the number line as another representation. • Facilitate the students to notice that when multiplying a rational number by a rational number that they need to convert the decimals to their fraction equivalents. • Support the students to notice the pattern of multiplying by ten.
Other Examples	<p>You can provide the answer to the first question to allow students to use their decimal and place value knowledge to solve the remaining equations.</p> <p style="text-align: center;"> $124 \times 54 = 6696$ $12.4 \times 54 =$ $1.24 \times 54 =$ $124 \times 0.54 =$ $0.124 \times 0.54 =$ </p>

PERCENTAGES: NUMBER STRING

$10\% \text{ of } 4.80 = 0.48$

$5\% \text{ of } 4.80 =$

$2.5\% \text{ of } 4.80 =$

$1.25\% \text{ of } \$4.80 =$

Big ideas	<p>A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.</p> <p>Percent is relative to the size of the whole.</p> <p>A percent is a special type of ratio where a part is compared to a whole and the whole is 100.</p> <p>A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.</p>
Curriculum Links	<ul style="list-style-type: none"> Recognise, read, write, represent, compare, order, and convert between fractions, decimals, and percentages
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Calculate percentages of (dollar) amounts, including decimals. Describe the strategies used using mathematical language.
Mathematical Language	Percentage, decimals, halving, division, multiplication.
Teacher Notes	<ul style="list-style-type: none"> The aim of this string is to show students that they can use benchmark percentages and place value knowledge to solve for harder percentages. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Show the students the first string asking; if we know ten percent, talk with your buddy how can we find 5%? Notice students who are able to explain their thinking explaining using the language of place value. Solve each equation before moving onto the next.
Other examples	<p style="text-align: center;">$10\% \text{ of } 156 = 15.6$</p> <p style="text-align: center;">$5\% \text{ of } 156 =$</p> <p style="text-align: center;">$2.5\% \text{ of } 156 =$</p> <p style="text-align: center;">$27.5\% \text{ of } 156 =$</p> <p style="text-align: center;">$10\% \text{ of } \\$18.99 =$</p> <p style="text-align: center;">$90\% \text{ of } \\$18.99 =$</p>

CALCULATE COSTS AND CHANGE FOR ANY AMOUNT OF MONEY.

\$185.95



Current New Zealand Banknotes.

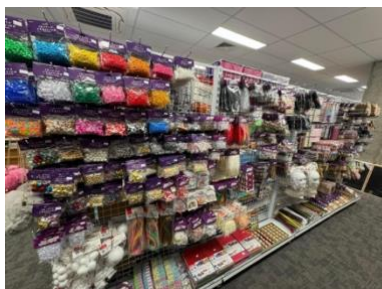
Photo credits: Nike, Investopedia websites

Which NZ bank notes could you use to buy this item?

Record different combinations and show how much change you would get?

Big ideas	<p>Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways.</p> <p>There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.</p>
Curriculum links	<ul style="list-style-type: none"> • In our number system, each place value is a power of 10, and this continues infinitely. • Represent whole numbers and decimals using powers of ten
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Explain and justify the use of place value to solve subtraction and problems.
Mathematical language	<p>Ones, tens, hundreds, thousands, add, subtract, difference, change total value, digit, addition, subtraction, banknotes,</p>
Teacher Notes	<p>The purpose of this activity is for students to Calculate costs and change using NZ bank notes.</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> 6. Place students in groups and encourage them to share their ideas with their buddies. 7. Encourage students to use a variety of representations to explain and justify their thinking eg. Empty number lines or PV houses. 8. Notice students using the inverse relationship between add/sub. Notice use of place value and the ability to see hundreds as ten tens and tens as ten ones. Draw connections to represent these within the place value houses. 9. Continue this activity with the students choosing items to add together and finding change.

WHERE IS THE MATHS?



What maths can you see in this photo?

What maths question could we ask?

(Note: use photos that will be engaging for your local community)

Big ideas	<ul style="list-style-type: none"> • The world is full of patterns and structures that we use mathematics and statistics to understand. • Mathematical practices are central to learning and doing mathematics.
Curriculum links	<ul style="list-style-type: none"> • Pose a question for investigation • Make connections with ideas in other learning areas and in familiar cultural, linguistic, and historical contexts.
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Form a mathematics question • Identify mathematics in everyday contexts
Mathematical language	Question, length, time, angle, amount, money, height, area ...
Teacher Notes	<p>This activity can cover all strands of the mathematics curriculum (number, measurement, algebra, space, statistics & probability).</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Display the picture and ask students to discuss in pairs what mathematics they see in the picture. 2. Share and collate all the ideas. Notice what students identify and ask a question that will extend their thinking. E.g., “where might <i>time/area/money</i> be in this photo?” 3. Ask “what maths question could we ask about this picture?” 4. Give students time to work with a partner to record questions. 5. Collect and share ideas. 6. To extend the activity, ask students to estimate (with reasoning) the answer to one of their questions.
Other examples	Use any photo, artwork or short video relatable to your students.

ALGEBRA | TAURANGI

TRUE OR FALSE STATEMENTS WITH SYMBOLS

$$6 + 8 > 15 - 12$$

$$(15 + 3) 2 < 1000 - 32$$

Are these equations true or false?

Justify your thinking.

Big ideas	Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained;
Curriculum links	<ul style="list-style-type: none"> • Look for patterns and regularities that can be applied in another situation or are always true. • Make and test conjectures, using reasoning and counterexamples to decide if they are true or not. • Use appropriate symbols to express generalisations.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Use correct symbols in equality equations. • Justify and explain their reasoning.
Mathematical language	Multiplication, division, problems, patterns, factors, sum, common factors, equal, multiplicand, multiplier, greater than, less than, equal to.
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Ask students in small groups to discuss what they notice. • Ask students to justify their thinking/explanations - Why is it true, false or equal? • Notice for students that are using arrows to justify their thinking across the equations • If needed remind students that = sign means the same on both sides/equal/balanced, > means greater than, < means less than. •
Other examples	$7 \times 2 > 3 \times 4$ $3 + 2 = \frac{13}{3} + 2$ $20 - 12 < 3 \times 3$ $\frac{1}{8} + \frac{3}{8} = \frac{1}{2}$ $15 - 12 < 19$ $14 < 14 - 1$ $15 > 15 - 13$

TRUE OR FALSE - INTEGERS

True or false?
Remember to justify your answers.

$$4 + 3 = -4 - 3$$

$$7 + 5 = 7 + -5$$

$$-3 + 6 = 6 + -3$$

Big ideas	<p>Equations show relationships of equality between parts on either side of the equal sign.</p> <p>The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained.</p> <p>Two quantities equal to the same third quantity are equal to each other. Mathematical situations can be represented as equations which include both positive and negative integers.</p> <p>A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.</p>
Curriculum links	<ul style="list-style-type: none"> • Solve simple addition and subtraction equations using integers. • Use a number line to represent the relationship between positive and negative integers in equations. • Explain, justify and represent reasoning related to maintaining equality between operations which involve integers.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Add or subtract using integers. • Justify their thinking.
Mathematical language	Integers, negative number, positive number.
Teacher Notes	<ul style="list-style-type: none"> • Select student solution strategies that use the properties of equality and understanding of negative numbers. • Highlight the difference between the use of - as an operation symbol (subtraction) and direction symbol (direction/size of movement) for negative numbers • Use equipment to show the idea of equality if needed • <u>Instructions</u>: Give students one of the equations to discuss with a partner before sharing back with the whole class.
Other examples	$4 + 3 = -4 - 3$ $40 + 30 = -40 - 30$ $400 + 30 = -400 - 300$

BALANCING EQUATIONS

$$158 + 45 = 57 + \underline{\quad}$$

$$47 + 46 + 45 = 37 + 36 + \underline{\quad}$$

Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. Equations show relationships of equality between parts on either side of the equal sign.
Curriculum Links	<ul style="list-style-type: none"> ● Solve open number sentences and true or false number sentences involving equality or inequality ● Use the distributive, commutative and associative properties
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Balance equations by finding relationships ● Explain the equals sign (=) represents balance
Mathematical language	Multiplication, multiply, groups of, factor, product, equals, commutative property, associative property, distributive property
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Reveal the equation ($158 + 45 = 57 + \underline{\quad}$). Ask students “what numbers might we put in the space to balance this equation?” 2. Encourage students to turn and talk about the products and to justify their reasoning. 3. Record all student solutions as they are shared as equations on the whiteboard. 4. Highlight solutions that draw on noticing a relationship between the left and right side of the equation (as opposed to calculating answers through trial and error). 5. Encourage students to discuss the second equation. Again focusing on the relationship across the equals sign.
Other examples	$33 + 44 = 33 + 22 + \underline{\quad}$ $456 + 789 = 466 + \underline{\quad}$ $98 - 78 = 88 - \underline{\quad}$ $35 - 12 = \underline{\quad} - 22$

BALANCING EQUATIONS (MULTIPLICATION AND DIVISION)

$$20 \times \underline{\quad} = 40 \times 16$$

Big Ideas	There are arithmetic properties that characterise addition and multiplication as operations. Equations show relationships of equality between parts on either side of the equal sign.						
Curriculum Links	<ul style="list-style-type: none"> ● Solve open number sentences and true or false number sentences involving equality or inequality ● Recall multiplication facts to 10 x 10 and corresponding division facts ● Use the distributive, commutative and associative properties 						
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Recall and apply multiplication facts ● Balance equations by finding relationships ● Explain the equals sign (=) represents balance 						
Mathematical language	Multiplication, multiply, groups of, factor, product, equals, commutative property, associative property, distributive property						
Teacher Notes	<p><u>Instructions:</u></p> <ol style="list-style-type: none"> 6. Reveal the first equation ($2 \times ? = 4 \times ?$). Ask students “what numbers might we put in the space to balance this equation?” 7. Encourage students to turn and talk about the products and to justify their reasoning. 8. Record all student solutions as they are shared as equations on the whiteboard. Discuss that there are multiple ways to balance this equation (e.g., $2 \times 60 = 4 \times 30$, $2 \times 5 = 4 \times 2.5$) 9. Highlight solutions that draw on noticing a relationship between the left and right side of the equation (as opposed to calculating answers through trial and error). 10. If students describe the equation $2 \times 4 = 4 \times 2$ then discuss the commutative property of multiplication. 11. Reveal the second equation $2 \times ? = 4 \times 16$. Give time for students to discuss what the missing factor is. 12. Use the questions as a discussion prompt to unpack the doubling and halving (proportional adjustment) relationship as the associative property of multiplication. 						
Other examples	<table style="width: 100%; border: none;"> <tr> <td style="padding: 5px;">$6 \times ? = 3 \times ?$</td> <td style="padding: 5px;">$5 \times ? = 10 \times ?$</td> <td style="padding: 5px;">$8 \times ? = 4 \times ?$</td> </tr> <tr> <td style="padding: 5px;">$6 \times ? = 3 \times 24$</td> <td style="padding: 5px;">$5 \times 50 = 10 \times ?$</td> <td style="padding: 5px;">$8 \times ? = 4 \times 19$</td> </tr> </table>	$6 \times ? = 3 \times ?$	$5 \times ? = 10 \times ?$	$8 \times ? = 4 \times ?$	$6 \times ? = 3 \times 24$	$5 \times 50 = 10 \times ?$	$8 \times ? = 4 \times 19$
$6 \times ? = 3 \times ?$	$5 \times ? = 10 \times ?$	$8 \times ? = 4 \times ?$					
$6 \times ? = 3 \times 24$	$5 \times 50 = 10 \times ?$	$8 \times ? = 4 \times 19$					

SOLVE SIMPLE EQUATIONS WITH VARIABLES

$$a + 12 = 13 + 6$$

Big ideas	<p>Three of more numbers can be grouped and added (or multiplied) in any order.</p> <p>If the same number is added or subtracted to both sides of an equation, equality is maintained.</p>
Curriculum links	<ul style="list-style-type: none"> • The commutative, associative, distributive, and identity properties work the same for all numbers. • A variable can be used to represent any number.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Use words and symbols to describe and represent the properties of operations (commutative, distributive, associative, inverse and identity). • Solve simple equations with variables
Mathematical Language	commutative, distributive, associative, equality,
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Remind students when looking at the equality in equations to look at the relationship across each side. • Place an equation on the board or screen. • Allow students time to think and then share their ideas with a buddy. • Notice students who are explaining their ideas to their buddies algebraically. • Use talk moves to facilitate participation and develop understanding when sharing back to the whole class. • Expect students to justify and explain their thinking when sharing back to their peers.
Other examples	<p>What's the value for each letter? Explain your thinking.</p> <ol style="list-style-type: none"> 1. $51 - a = 26$ 2. $15 \times a = 2 \times 1.5 \times 10$ 3. $64 = 8 \times 2 \times a$ 4. $125 = 5 \times 5 \times a$

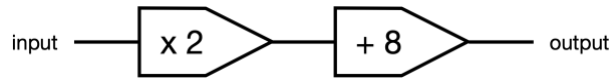
TRUE OR FALSE – EXPANDING BRACKETS

Mele thinks that $12(2b-3)$ is equivalent to $24b - 3$.

Do you agree with Mele? Why or why not?

Big ideas	Algebraic expressions can be named in an infinite number of different but equivalent ways (e.g., $2(a - 12) = 2a - 24 = 2a - (28 - 4)$).
Curriculum links	<ul style="list-style-type: none"> • A variable can be used to represent any number.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Solve linear equations by trial and improvement and by applying inverse operations. • Reason with algebraic thinking.
Mathematical Language	Inverse operations, linear equations, variable
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Show the equation. • Give students the time to think. Allow students to talk to a buddy about their thinking and ideas. • Notice students who are able to reason algebraically discussing the equivalence on both sides. • Expect students to explain and justify their thinking when students are sharing back. • Use talk moves to facilitate participation and develop understanding. • Represent student ideas on the whiteboard.
Other examples	<p style="text-align: center;">True or False. Why?</p> <p style="text-align: center;">$4(2a + 3) = 8a + 3$</p> <p style="text-align: center;">$6(2a - 6) = 12a - 36$</p> <p style="text-align: center;">$4(8a - 4) = 2a - 1$</p> <p style="text-align: center;">1. Kiriwai thinks that $6(2b-3)$ is equivalent to $12b - 18$. Do you agree with Kiriwai? Why or why not?</p>

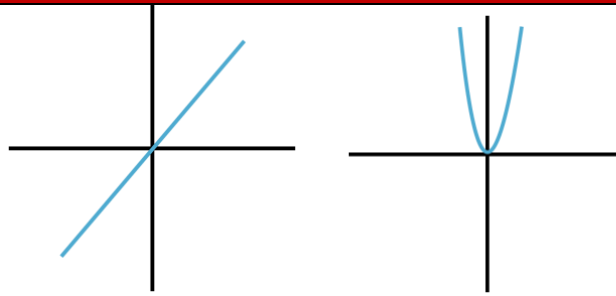
FUNCTION MACHINE



Run 3 consecutive numbers through the function machine.
What do you notice?
What would the algebraic rule be?

Big ideas	Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	<ul style="list-style-type: none"> Use tables, XY graphs, and diagrams to find relationships between elements of growing patterns. Develop a rule in words about a linear pattern.
Learning Outcomes: Students will be able to:	<ul style="list-style-type: none"> Identify the element for a repeating pattern for far terms. Explain that a pattern has consistency. Develop a rule for a function machine and express it in words.
Mathematical language	Constant, unit of repeat, rule, sequence, variable, function, machine, input, output, multiply, add,
Teacher Notes	<ul style="list-style-type: none"> Ensure that prior to this warmup, your class has had exposure to patterns and algebraic rules. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Show students the function machine. Ask the students to choose their own numbers within their peers to enter through the machine. Encourage the students to explore the function machine before unpacking 'how to'. Notice students who are able to explain 'how to' using mathematical language. When sharing back with the whole class use talk moves to highlight key mathematical ideas. Prompt for the algebraic rule if necessary, representing on the board for all to see; outcome = number $\times 2 + 8$, so $y = 2x + 8$.
Other examples	<div style="margin-bottom: 10px;"> </div> <div> </div>

ALGEBRAIC GRAPHS



What graph represents $y = x$?
Explain and justify your thinking.

Big ideas	They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula and then graphed.
Curriculum links	<ul style="list-style-type: none"> • Use tables, XY graphs, and diagrams to find relationships between elements of growing patterns. • Develop a rule in words about a linear pattern.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Identify the graph that matches the algebraic rule. • Justify their algebraic reasoning.
Mathematical language	Constant, unit of repeat, rule, sequence, variable, function, machine, input, output, multiply, add,
Teacher Notes	<ul style="list-style-type: none"> • Ensure that prior to this warmup, your class has had exposure to patterns and algebraic rules. • This starter is designed for students to explore and have exposure to algebraic graphs. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Show students the graphs. • Encourage students to turn and talk with their peers. What do they notice? What could the variables be? • Notice students that are using algebraic language to explain their thinking. • If needed prompt students to give y a value.. If $y = 1$, then $x = 1$ etc... if these were graphed what would it look like? • Prompt students to think about the other graph what could the algebraic rule be.
Other examples	Use www.desmos.com/calculator to create other graphs that can be discussed and compared.

MEASUREMENT – INE

CONVERSIONS OF MEASUREMENTS

Which measurement is the greatest?

Distance to the River	
THIS	THAT
34.7km	3470m
$3\frac{2}{5}$ km	34000m
0.37 km	34.7m

Explain and Justify your reasoning.

Big Ideas	There are a range of attributes that we can measure including length, mass, time, area, angle, and volume. When we measure, we use comparison, specifically, we compare like properties to see which is greater. We can make comparisons using standard or non standard units of measure and we use mathematical language to describe these.
Curriculum Links	<ul style="list-style-type: none"> • In the metric system, there are base measurements with prefixes added to show the size of units. • Metric measurements can be converted from fractions to whole numbers, and vice versa, by changing units.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • convert between units of measurement • justify and explain their reasoning
Mathematical language	Unit of measure, measurement count, convert, millimetre, centimetre, metre, kilometre, millilitre, litre, milligram, gram, kilogram, gigabytes, megabytes, hours, minutes, seconds
Teacher Notes	<ul style="list-style-type: none"> • In this activity, students decide which option is the greater amount (this or that). <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • You can show one comparison at a time or all three at once. • Provide time for students to reason and compare the measurements, and to develop a justification for their answer. • Students can turn and talk and share their thinking with a partner. Encourage the use of agree or disagree and why with their partner. • Facilitate a large group discussion about different reasonings students had as they share their justifications. <p>Questions to support discussion:</p> <ul style="list-style-type: none"> • How did you convert the units? • How could we work out what the difference is between the two?

- What did you multiply/divide by to compare the measurements and why?
- Can you use decimals or fractions to give different representations of the same measurement?

Other examples

Amounts can be changed to decimal and fractional numbers to increase challenge.

Amount of milk left in the bottle

THIS	THAT
200.7ml	0.27l
$3\frac{2}{3}$ l	3230ml
65.05ml	0.65l

Length of Lego bricks

THIS	THAT
555 mm	0.55m
$3\frac{1}{4}$ cm	35mm
75mm	0.07m

Weight of apple bins

THIS	THAT
50.48kg	4800g
45005g	$45\frac{1}{5}$ kg
380kg	308000g

Gigabytes of Data

THIS	THAT
550mb	0.5GB
$10\frac{1}{2}$ GB	1500MB
28000MB	24.5GB

ESTIMATING AREA AND PERIMETER

Can you estimate the area and perimeter of this shape?
Turn and talk to your buddy. Remember to explain and justify your thinking.



Can you convert your estimations to metres?

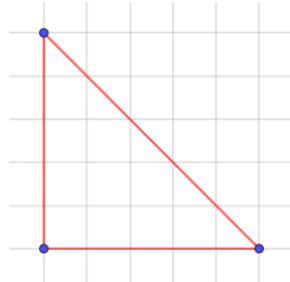
Big ideas	<p>There are a range of attributes that we can measure including length, mass, time, area, angle, and volume. When we measure, we use comparison, specifically, we compare like properties to see which is greater.</p> <p>We can make comparisons using standard or nonstandard units of measure and we use mathematical language to describe these.</p>
Curriculum links	<ul style="list-style-type: none"> • estimate and then measure length, area, volume, capacity, mass, temperature, data storage, time, and angle, using appropriate metric units. • visualise, estimate, and find the perimeter and area of shapes composed of triangles and rectangles.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Estimate and measure length in a range of measurement units (mm, cm, m). • Identify the relationship between millimetres, centimetres and a metre.
Mathematical language	Metre, centimetre, millimetre, length, unit of measure, measurement count, ruler.
Teacher Notes	<ul style="list-style-type: none"> • Please do not label the sides of the shape prior to students solving the starter. Direct student to noticing that each box is 1 cm. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Place the starter on the board and encourage students to share their thinking with a buddy. • Notice for students who break the shape into compound shapes and use the area formula. • Students may also count the number of squares in the shape to find area. • Select students to share back that have clear mathematical explanations. • Use talk moves to engage other students in the conversation and to support participation.

THE PERIMETER ISWHAT IS THE AREA?

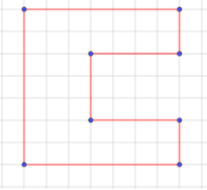
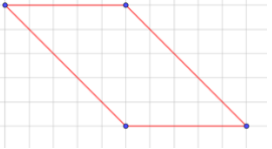
The perimeter is 22cm. What is the area?

Big ideas	Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways. Some attributes of objects are measurable and can be quantified using unit amounts.
Curriculum links	<ul style="list-style-type: none"> • Metric measurements can be converted from fractions to whole numbers, and vice versa, by changing units. • Shapes can be decomposed or recomposed to help us find perimeters, areas, and volumes.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ● Use non-standard units (squares) to measure area. ● Find the area of a surface by using multiplication. ● Develop a generalisation for finding the area of a rectangle. ● Use measurement language to describe how to measure area.
Mathematical language	Area, square, unit of measure, measurement count.
Teacher Notes	<ul style="list-style-type: none"> • The objective is for students to explore the idea that objects can have the same perimeter however a different area. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Decide on the shape for the task (quadrilateral) or alternatively let students choose their own. • Pose the questions and then support students to discuss with their group. • Encourage the students to think of decimal options as well or • Notice for students that are using the language of measurement when sharing their ideas to their peers. • Use talk moves to facilitate the classroom discussion and support participation. • For example: $1\text{cm} \times 10\text{ cm} = 10\text{cm}^2$ or $.5\text{cm} \times 10.5\text{cm} = 5.25\text{cm}^2$. • Encourage students to notice the pattern in the different options.
Other examples	<p>The perimeter is .75m what is the area?</p> <p>The perimeter is 90cm what is the area?</p>

EXPLORING AREA



How many ways can you split this shape into two parts that have the same area?

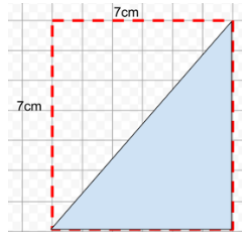
Big ideas	<p>There are a range of attributes that we can measure including length, mass, time, area, angle, and volume. When we measure, we use comparison, specifically, we compare like properties to see which is greater.</p> <p>We can make comparisons using standard or nonstandard units of measure and we use mathematical language to describe these.</p>
Curriculum links	<ul style="list-style-type: none"> • Estimate and then measure length, area, volume, capacity, mass, temperature, data storage, time, and angle, using appropriate metric units. • Visualise, estimate, and find the perimeter and area of shapes composed of triangles and rectangles.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Make two shapes with equal areas • Recognise square units
Mathematical language	Area, length, height, cm^2 , $\frac{1}{2} \text{cm}^2$
Teacher Notes	<p>Provide access to grid paper.</p> <p>Instructions:</p> <ul style="list-style-type: none"> • Notice students who are counting in unit squares or applying a rule to find the total area of this triangle (12.5cm^2). • Explore different ways the total area can be partitioned into two parts, shapes that have an area of 6.25cm^2
Other examples	<div style="display: flex; justify-content: space-around; align-items: center;">   </div> <p>Repeat with a range of other regular and compound shapes. You can also change the number of parts. E.g., can you split this shape into three parts with the same area?</p>

EXPLORING AREA OF RIGHT ANGLE TRIANGLES

Lauren and Emily make the claim that they can find the area of this triangle by finding the area of the rectangle that this triangle fits inside, and then halving it.

Here are their workings:

Area of rectangle:
 $7 \times 7 = 49\text{cm}^2$
 Area of triangle:
 $49 \div 2 = 24.5\text{cm}^2$

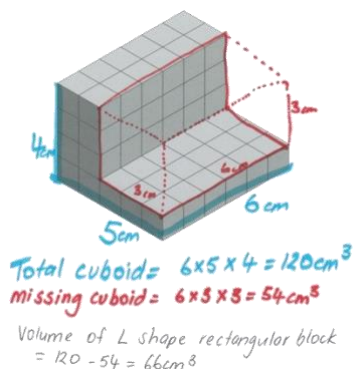


Big ideas	<p>There are a range of attributes that we can measure including length, mass, time, area, angle, and volume. When we measure, we use comparison, specifically, we compare like properties to see which is greater.</p> <p>We can make comparisons using standard or nonstandard units of measure and we use mathematical language to describe these.</p>
Curriculum links	<ul style="list-style-type: none"> • Estimate and then measure length, area, volume, capacity, mass, temperature, data storage, time, and angle, using appropriate metric units. • Visualise, estimate, and find the perimeter and area of shapes composed of triangles and rectangles.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Find area of right angle triangles • Recognise square units
Mathematical language	Area, length, height, cm^2 , $\frac{1}{2} \text{cm}^2$
Teacher Notes	<p>Provide access to grid paper.</p> <p>Instructions:</p> <ul style="list-style-type: none"> • Notice students who are counting in unit squares or applying a rule to find the total area of this triangle. • Students may fold and cut the paper to compare triangles to prove if this strategy works. • Listen for mathematical argumentation and justification and use talk moves to have students share their explanations.
Other examples	<p>Repeat with a range of other triangles. Start with right angle triangles and then progress to different types of triangles. How can they prove if this strategy still works on other triangles?</p>

CALCULATING VOLUME

Annah makes the claim that she can find the volume of an L shaped rectangular block by converting it into a cuboid and finding the total volume, and then subtracting the extra volume that was added to create the cuboid.

Here are her workings:



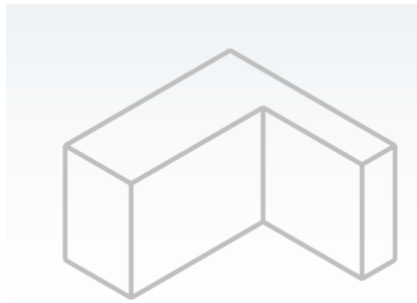
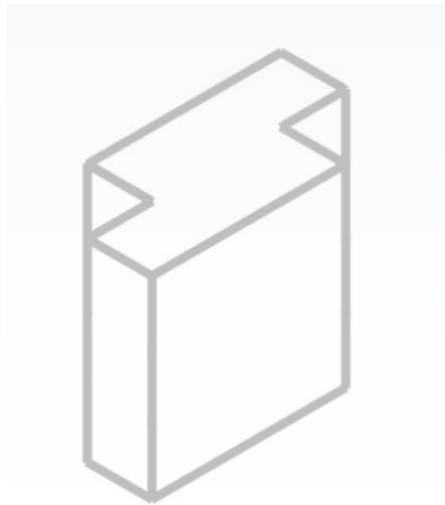
Can you explain what she has done in her own words? Do you agree that this works? Why or why not does this give an accurate measure of volume for an L shaped block? How could you test Annah's claim?

Big ideas	<p>There are a range of attributes that we can measure including length, mass, time, area, angle, and volume. When we measure, we use comparison, specifically, we compare like properties to see which is greater.</p> <p>We can make comparisons using standard or nonstandard units of measure and we use mathematical language to describe these.</p>
Curriculum links	<ul style="list-style-type: none"> • Use appropriate scales, devices, and metric units for length, area, volume and capacity, weight (mass), temperature, angle, and time. • Use side or edge lengths to find the perimeters and areas of rectangles, parallelograms, and triangles and the volumes of cuboids. • Use a range of multiplicative strategies when operating on whole numbers.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ∉ Use multiplicative reasoning to find volume. ∉ Find the volume of a composite cuboid.
Mathematical language	Cuboid, length, breadth, height, surface, centimetre, 3-dimensional, 2-dimensional, volume, width, depth, rectangular prism, dimensions.
Teacher Notes	<ul style="list-style-type: none"> • Expect students to understand that volume is the measurement of space and therefore the volume of the “missing space” can also be calculated. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Show students Annah's workings and tell students to discuss the claim with a buddy.

- Pay attention to any mathematical argumentation and highlight this with the larger group. Who thinks the same, who thinks differently and why? Who has changed their thinking and why? What convinced you?
- Select several students to share how they could check if this claim is true.
- Ask students if they think this strategy would work on other shapes? Why or why not?

Other examples

You may choose to use this warm up over several days. Show students other examples and ask them how they could use this strategy to calculate the volume of these shapes.



SCALE FACTOR – VOLUME

Using a small unit cube. If the cube is enlarged by a factor (f) of 2. What scale factor is the volume enlarged?

Big ideas	Relationships between scale factors for length, area, and volume is conceptually difficult to understand. The notion of scale factors for lengths areas and volume. The relationship between scale factors for length, area and volume.
Curriculum links	<ul style="list-style-type: none"> • Scale a shape by a factor and then derive the scale factor for the new shape’s area or volume
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Use scale factors to investigate areas being enlarged. • Use scale factors to investigate volumes being enlarged. • Solve real life context problems involving scale factors.
Mathematical language	scale factors, surface area, volume, enlarged
Teacher Notes	<ul style="list-style-type: none"> • Facilitate discussion using the small unit cubes. • Make available a pile of small cubes so that students can build up cubes that are twice and three times the size, and then count the new surface areas and volumes. • Expect students to justify their thinking when sharing statements to the classroom. Provide opportunities for students to reason with their peers thinking by asking if they agree or disagree. • Possible misconception is students calculate the volume by length x width x height and multiplying by 2.
Other examples	Extend this task by asking the students to represent their enlargements.

CONVERT DIGITAL TO ANALOGUE TIME

This is a time on a digital clock.

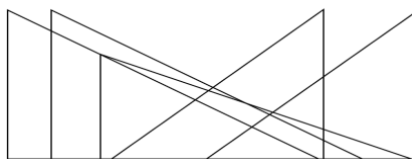
13:18

What analogue time is it? Prove your thinking.

Big ideas	<p>There are multiple ways to measure time and some units of time measurement are more appropriate than others within different contexts.</p> <p>Time is displayed in different ways depending on the context. Numbers that are used to measure time repeat themselves in a cycle.</p> <p>Time measurements can be compared when they are converted into the same unit.</p>
Curriculum links	<ul style="list-style-type: none"> • Convert between measurement units • Read analogue and digital measurement tools, round appropriately, and interpret scales accurately
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Convert between digital and analogue time • Explain and justify my thinking
Mathematical language	<p>Morning, afternoon, evening, night, day, tomorrow, yesterday, after, before, longer, shorter, equal, seconds, minutes, hours, week, month, year, decade, time, measurement, timeline, midday, midnight, noon, analogue clock, digital clock, clockwise, anticlockwise, circular numberline, circumference, intervals, quarter hour, half an hour, three quarters of an hour, duration</p>
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Allow students to represent the time by drawing clocks if needed. • As students are discussing their ideas with a partner, notice for students that are using mathematical language to explain their thinking. • When sharing back ideas to the whole group, encourage students to justify their thinking i.e. explain why.
Other examples	<p>Use a variety of other times to consolidate this skill.</p> <ul style="list-style-type: none"> ❖ 19:45 ❖ 02:30 ❖ 18:00 ❖ 03:10 ❖ 04:45

SPACE MOKOWĀ

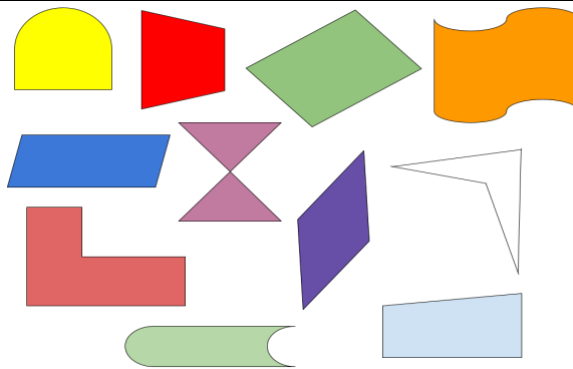
TRIANGLES



Can you name and explain the different triangles found in the shape above?

Big Ideas	<p>Two-and-three dimensional objects with or without curved surfaces can be described, classified, and analysed by their attributes.</p> <p>Shapes have sides that are parallel, perpendicular, or neither.</p> <p>Shapes have line symmetry, rotational symmetry, or neither.</p> <p>Shapes are similar, congruent, or neither.</p>
Curriculum links	<ul style="list-style-type: none"> Classify shapes based on their geometric properties.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Identify classes of shapes in a range of different ways using geometrical language to explain and justify.
Mathematical language	<p>Properties, shape, rectangle, square, side, equal, straight, parallel, angles, size, edge, corner, vertices, congruent, symmetrical, curved, triangle, quadrilateral, hexagon, square corner, right angle, rhombus, parallelogram, kite, trapezoid, regular, irregular, pentagon, hexagon, heptagon, octagon, equilateral, acute angle, obtuse angle.</p>
Teacher Notes	<ul style="list-style-type: none"> Note: This starter might be done over a few days, or as a starter and then finished as an independent activities (provide students with the opportunity to write clear definitions for each triangle). <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Show the image and provide the students some time to discuss with a partner. Notice for students who are using geometrical language to explain and justify their thinking. Triangles can be classified by their sides or angle: <ul style="list-style-type: none"> Equilateral Triangle: three equal sides Isosceles triangle: two equal sides Scalene triangle: no equal sides Acute Triangle: has three right angles Right Angle Triangle: Has one 90° angle Obtuse Triangle: has one angle $> 90^\circ$ When sharing back student ideas use talk moves to highlight key mathematical reasoning. Further points of discussion? <ul style="list-style-type: none"> Can triangles be classified by both angle and side, e.g. a Obtuse Isosceles Triangle? What irregular shapes can you find in this image?
Other examples	<p>Explore and justify shapes in the environment around you.</p>

IS IT A QUADRILATERAL?



Are these shapes quadrilaterals? Justify your thinking.

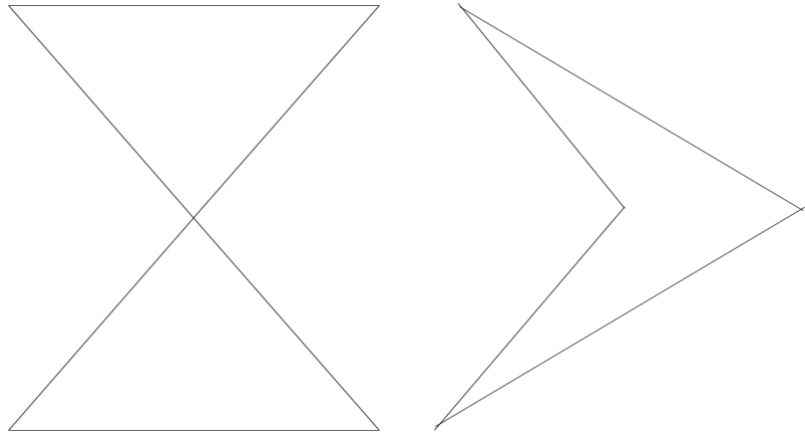
Big ideas	Two-and-three dimensional objects with or without curved surfaces can be described, classified, and analysed by their attributes.
Curriculum links	∉ Sort and classify plane shapes into classes and sub classes according to defined geometrical properties
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ∉ Identify classes of shapes in a range of different ways using geometrical language to explain and justify. ∉ Use commonly shared rules to communicate ideas about defining shapes.
Mathematical language	2-dimensional, straight, collinear, angles, vertices, vertex, sides, vertical, horizontal, diagonal, symmetrical, face, curved, edge, corner, triangle, quadrilateral, diamond, kite, trapezoid, rhombus, rectangle, square, parallelogram, square corner, right angle, regular, irregular, pentagon, hexagon, heptagon, octagon, equilateral, scalene, acute angle, obtuse angle.
Teacher Notes	<ul style="list-style-type: none"> ● A quadrilateral is a 2D shape that has four straight sides and four corners. Quadrilaterals can be grouped based on their properties. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> ● Show students the 2D shapes. Emphasise the need to justify their reasoning. This shape is not a quadrilateral because.... ● Pay attention to any mathematical argumentation and highlight this with the larger group. Who thinks the same, who thinks differently and why? Who has changed their thinking and why? What convinced you? ● Questions to further support discussion: <ul style="list-style-type: none"> - What do you notice about number of sides, length of sides, number of parallel lines, size of angles?
Other examples	You may choose to use one shape per day and use this warm up over a series of days or independent tasks. Use materials within your environment when available.

This activity can be further extended by looking at different quadrilaterals and seeing if they can be classed as more than one type of quadrilateral. For example, a square is a rectangle, is a rectangle also a square? Can a rhombus also be a parallelogram? Why or why not?

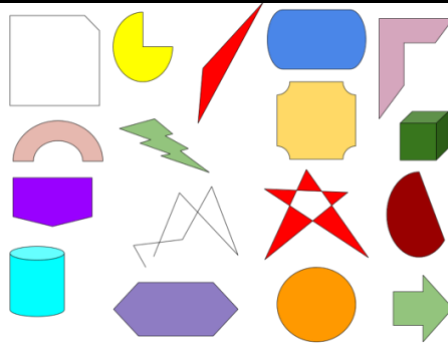
Show students a hierarchy of quadrilaterals and get them to come up with their own descriptions of each type of quadrilateral based on length of sides, number of parallel lines, size of angles.

Note: The following shapes are quadrilaterals:

Intersecting quadrilateral and concave quadrilateral. Why are these shapes classified as quadrilaterals?



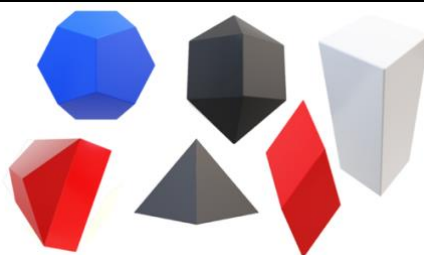
IS IT A POLYGON?



Are these shapes polygons? Justify your thinking.

Big ideas	Two-and-three dimensional objects with or without curved surfaces can be described, classified, and analysed by their attributes.
Curriculum links	≠ Sort and classify plane shapes into classes and sub classes according to defined geometrical properties
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ≠ Identify classes of shapes in a range of different ways using geometrical language to explain and justify. ≠ Use commonly shared rules to communicate ideas about defining shapes.
Mathematical language	2-dimensional, straight, collinear, angles, vertices, vertex, sides, vertical, horizontal, diagonal, symmetrical, face, curved, edge, corner, triangle, quadrilateral, diamond, kite, trapezoid, rhombus, rectangle, square, parallelogram, square corner, right angle, regular, irregular, pentagon, hexagon, heptagon, octagon, equilateral, scalene, acute angle, obtuse angle.
Teacher Notes	<ul style="list-style-type: none"> ● A polygon is a 2D closed shape with three or more straight sides. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> ● Show students the 2D shapes. Emphasise the need to justify their reasoning. This shape is not a polygon because.... ● Pay attention to any mathematical argumentation and highlight this with the larger group. Who thinks the same, who thinks differently and why? Who has changed their thinking and why? What convinced you? ● Questions to further support discussion: <ul style="list-style-type: none"> - What do you notice about number of sides, length of sides, number of parallel lines, size of angles? - Introduce regular and irregular polygon terminology. Ask students what they think this means and which shapes do they think belong in each category.
Other examples	You may choose to use one shape per day and use this warm up over a series of days or independent tasks. Use materials within your environment when available.

IS IT A PRISM?

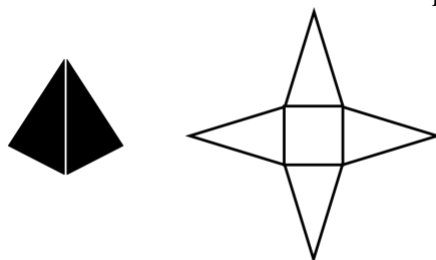


Is it a prism? Justify your thinking.

Big ideas	Two-and-three dimensional objects with or without curved surfaces can be described, classified, and analysed by their attributes.
Curriculum links	<ul style="list-style-type: none"> ∉ Sort and classify plane shapes into classes and sub classes according to defined geometrical properties
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> ∉ Identify classes of shapes in a range of different ways using geometrical language to explain and justify. ∉ Use commonly shared rules to communicate ideas about defining shapes.
Mathematical language	2-dimensional, 3- dimensional, straight, collinear, angles, vertices, vertex, sides, vertical, horizontal, diagonal, symmetrical, face, curved, edge, corner, sphere, cylinder, cube, cuboid, rectangular prism, triangle, quadrilateral, equilateral triangle, square corner, right angle, regular, irregular, pentagon, hexagon, heptagon, octagon, equilateral, scalene, acute angle, obtuse angle.
Teacher Notes	<ul style="list-style-type: none"> ● A prism has two congruent, parallel polygon shaped bases facing each other. The bases are connected by rectangular or parallelogram-shaped sides. The number of sides on each base defines the type of prism. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> ● Show students the 3d shapes. Emphasise the need to justify their reasoning. This shape is not a prism because.... ● Pay attention to any mathematical argumentation and highlight this with the larger group. Who thinks the same, who thinks differently and why? Who has changed their thinking and why? What convinced you? ● Questions to further support discussion: <ul style="list-style-type: none"> -What would a cross section of a prism look like in different places? -How is this different to a 3d shape that is not a prism?
Other examples	You may choose to use one shape per day and use this warm up over a series of days or independent tasks. Use materials within your environment when available.

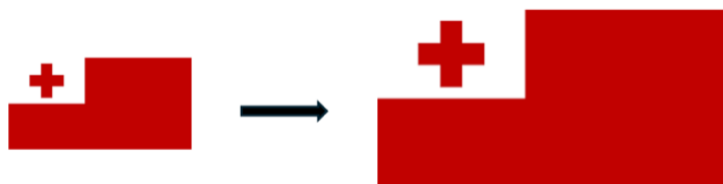
IS THIS THE CORRECT NET FOR THIS 3D SHAPE?

Is this the correct net for this 3d shape?



Explain and justify how you know.

INVARIANT PROPERTIES



Look at this transformation.


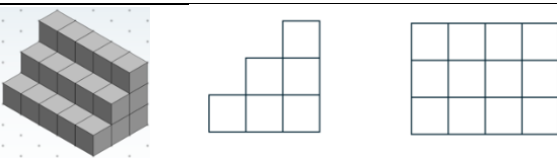
What properties have stayed the same? What properties have changed?

Big ideas	Objects in space can be transformed in an infinite number of ways, and those transformations can be described and analysed mathematically.
Curriculum links	<ul style="list-style-type: none"> The invariant properties of two- and three-dimensional shapes do not change under different transformations.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Identify variant and invariant properties of a shape under transformation Analyse a transformation mathematically
Mathematical language	Angle, area, length, width, shape, position, orientation, enlargement, transformation, translation, reflection, rotation, invariant, variant
Teacher Notes	<p>Invariant properties do not change (e.g., the angle of each corner remains 90°, orientation remains the same). Variant properties do change (e.g., area increases, length increases)</p> <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Show students the image and allow sufficient time for students to discuss what properties are changing and staying the same. Listen for students who are analysing the shape mathematically (e.g., discussing angles, area, orientation, length, width...). Encourage students to share their ideas. Press towards developing a claim about which properties will be invariant when a shape undergoes enlargement. Reinforce the concept that invariant properties are those that do not change under transformation.
Other examples	<p>Repeat this task for translations, rotations & reflections. Keep the focus on analysing the invariant and variant properties. E.g.,</p> <div style="text-align: center;"> </div>

USING PLAN VIEWS



Here are the plan views. What might the 3-dimensional building look like?

Big ideas	Objects in space can be oriented in an infinite number of ways, and an object's location in space can be described quantitatively.
Curriculum links	<ul style="list-style-type: none"> • Use plan-view drawings to visualise and construct three-dimensional shapes • Three-dimensional shapes can be represented by two-dimensional images.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Use 2D plans to create a 3D shape. • Represent a 3D from various viewpoints. • Explain an image using mathematical language.
Mathematical language	3D shape, plan-view, top-down, left-side, front-side, right-side, length, height, elevation, depth, width, square, cube
Teacher Notes	<p>These plan views were drawn from this 3D shape.</p>  <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Students will need access to isometric grid paper/ multilink cubes/ or an online tool. • Present the image and give sufficient time for students to explore creating a building that will align with the dimensions of the given plans. • Expect students to justify their 3D building using mathematical language (height, length, units)
Other examples	 <p>Explore giving students the 3D model and asking them to draw it from a particular viewpoint (e.g., top-down, right-side). Or continue with providing more complex view plans, with students constructing the 3D model.</p> <p>https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Isometric-Drawing-Tool/</p>

GEOMETRY: WHAT ANGLES DO YOU NOTICE?

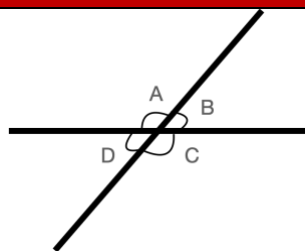


(Artist: Tui Emma Gillies, 2023)

What angles do you notice?

Big ideas	Objects in space can be oriented in an infinite number of ways, and an object's location in space can be described quantitatively.
Curriculum links	Position , direction, and pathways can be described using te ao tūroa, as in Māori and Pacific systems of knowledge, or using scale, compass points, and environmental features.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Find unknown angles and identify angle properties of intersecting lines • Talk about angles formed by two intersecting lines in the plane are related in special ways (e.g., vertical angles) • Notice when a line intersects two parallel lines the angles formed are related in special ways.
Mathematical Language	Acute, Obtuse, Reflex, Right Angle, Straight Line Angle, Complete Angle
Teacher Notes	<ul style="list-style-type: none"> • Key question to ask students “What do you notice?” “What angles do you notice on the picture?” • Use talk moves to facilitate participation and develop understanding. • Discuss what students notice. Teacher to facilitate and emphasise the angles that students notice and discuss their properties.
Other examples	<div style="text-align: center;"> </div> <p style="text-align: center;">(Artist: Tui Emma Gillies, 2017)</p> <p>Or use images from around the community.</p>

INTERSECTING ANGLES



Hamish believes we only need to know the measure of one of these angles to calculate the rest.

Do you agree or disagree with this mathematical claim?

Big ideas	Relationship between the connecting rays that constitutes the angle. That relationship is the turning of one ray onto the other, at about the point where they meet.
Curriculum links	<ul style="list-style-type: none"> describe an angle using the benchmarks 90 degrees, 180 degrees, and 360 degrees. find unknown angles and identify angle properties of intersecting lines
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Identify angles. Explain angles using mathematical language Justify their mathematical reasoning
Mathematical language	Turn, angle, degrees, rotation, right, acute, obtuse, parallel, properties of lines, adjacent angle, intersecting, corresponding angles
Teacher Notes	<ul style="list-style-type: none"> This starter is designed to encourage students to reason and state clear mathematical justifications and explanations about angles. The angle properties of lines are: <ul style="list-style-type: none"> Vertically opposite angles are equal (a and c or b and d) Adjacent angles add to 180° (a and b) <p><u>Instructions:</u></p> <ul style="list-style-type: none"> Pose the mathematical claim to the students and ask them to turn and talk with a partner to share their ideas. Notice for students who are able to use correct terminology when justifying their thinking. When sharing back ideas to the class, use talk moves to highlight key mathematical ideas demonstrated. Expect students to give the angles values when justifying their thinking.
Other examples	Explore this idea using shapes. Does this claim work on all intersecting lines?

LATITUDE AND LONGITUDE

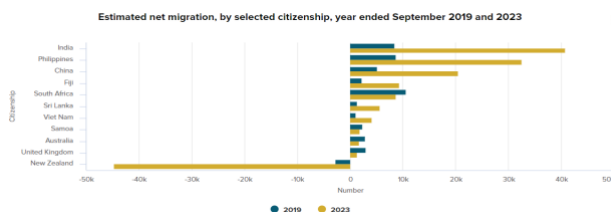


How could you describe the location of Wellington using this map?

Big ideas	A geographic coordinate system is a spherical or geodetic coordinate system for measuring and communicating positions directly on the Earth as latitude and longitude.
Curriculum links	<ul style="list-style-type: none"> Use scale, compass points, and coordinate systems to interpret and describe positions and pathways.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> Describe the location of a landmark using latitude and longitude.
Mathematical Language	Latitude, longitude, compass directions (N,S,E,W), degrees
Teacher Notes	<p><u>Instructions:</u></p> <ul style="list-style-type: none"> Have available a globe or large world map. Students to discuss how they would describe the location of Wellington. Listen for students who are using compass directions/latitude/longitude in their explanations. E.g., “Wellington is almost sitting on the 175°E latitude line.” Share explanations and annotate evidence on the map. Press students to compare the location of Wellington to the latitude and longitude lines. Ask students “why might it be important to have an accurate way to give locations of places in the world?”
Other examples	Describe the location of any country/city/landmark of interest to students using a variety of world maps or globes.

STATISTICS - TAUANGA

BAR GRAPH STARTER

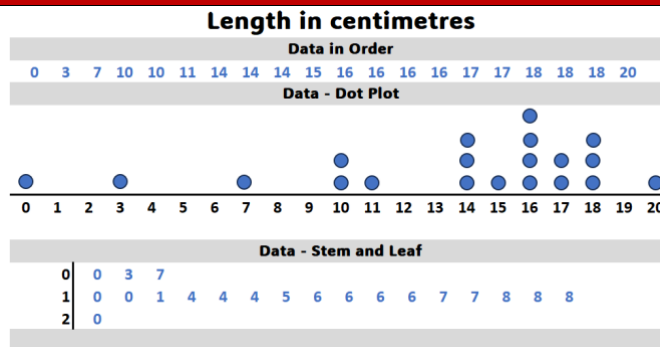


Ref: <https://www.stats.govt.nz/information-releases/international-migration-september-2023/>

What do you notice?

Big ideas	<p>Ideas and questions about a specific topic can be investigated through collecting data and using it to answer the questions. Data can vary in different ways (e.g., an object can be different sizes and colours) and it can be organised in different ways and by different characteristics (categorical, numerical). Data can be represented and communicated in multiple ways including data visualisations.</p>
Curriculum links	<ul style="list-style-type: none"> analyse data and communicate findings in context examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are believable and reasonable.
Learning Outcomes: Students will be able to:	<ul style="list-style-type: none"> make statements and give explanations inductively based on observations or data recognise and explore patterns, and make conjectures and draw conclusions about them
Mathematical language	<p>Statistics, data, sample, investigate, organise, display, sort, classify, represent, communicate, predict, outcomes, stem-and-leaf graph, mode, median, range, cluster, outlier.</p>
Teacher Notes	<p>Instructions</p> <ul style="list-style-type: none"> Show the bar graph on the board. Give students time to reflect on the data and then ask them what they notice about the data. Building on their observations ask them: <ul style="list-style-type: none"> <i>What is the data telling us?</i> <i>From which country did the largest group of immigrants come from?</i> <i>Which country saw a drop in migration to NZ?</i> <i>Why might this data have been collected?</i>
Other examples	<p>A further question might be: <i>What reasons can you think of for the changes in migration between 2019 and 2023?</i></p>

GRAPHS - WHAT DO YOU NOTICE?



What do you notice about the graphs?

Do both graphs represent and show the same and different information.

Explain and justify why?

Big ideas	<p>A graph for displaying the distribution of a numerical variable in which each dot represents each value of the variable.</p> <p>Dot plots are particularly useful for comparing the distribution of a numerical variable for two or more categories of a category variable by displaying side-by-side dot plots on the same scale. Ideally the numbers in the ‘stem’ represent the highest place-value digit in the values and the ‘leaves’ display the second highest place-value digits in each individual value.</p>
Curriculum links	<ul style="list-style-type: none"> • Data visualisations show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Describe the data using dot plot and stem and leaf • Interpret information from graphs and make statements about them
Mathematical language	Graphs, dot plot, stem and leaf, distribution, cluster, outliers
Teacher Notes	<ul style="list-style-type: none"> • Encourage students to make statements about the graphs and how could the graphs be the same and different. • What is the same and difference between a dot plot and stem and leaf. • The graph(s) show the variable of distance in cm’s. Distance is a discrete variable. • Students are to understand both graphs could be used to highlight and describe the idea of clustering. A “cluster” is a group of data found together in a clump. • Support students to identify outliers, decide if values in the data and graph are at the extremes.
Other examples	Use statistics NZ to source graphs to discuss.

STEM AND LEAF STARTER

Time spent per month

On Screens (e.g. Computer or TV)		Playing Outside
Leaf	Stem	Leaf
	0	2 3 5 6 6 8 9
6 5 4 3 1 0	1	0 2 5 5 7 7 8 9
9 8 7 5 4 3 2 0 0	2	1 2 6 8 9
6 5 4 4 2	3	0
6 5 5 5	4	2 3
	5	
	6	6
	7	
	8	

What does this graph tell us?

Big ideas	Data can vary in different ways (e.g., an object can be different sizes and colours) and it can be organised in different ways and by different characteristics (categorical, numerical). Predictions can be made through using sets of data.
Curriculum links	<ul style="list-style-type: none"> analyse data and communicate findings in context examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are believable and reasonable.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> make statements and give explanations inductively based on observations or data recognise and explore patterns, and make conjectures and draw conclusions about them
Mathematical language	Statistics, data, sample, investigate, organise, display, sort, classify, represent, communicate, predict, outcomes, stem-and-leaf graph, mode, median, range, cluster, outlier.
Teacher Notes	<p><u>Instructions</u></p> <ul style="list-style-type: none"> Show the stem and leaf graph on the board. Give students time to reflect on the data and then ask them what they notice about the data, what is the graph telling us? Notice students using statistical language in their conversations. Building on their observations ask them: <i>What is the data telling us?</i> <i>What activity is shown in the graph as being more popular?</i> <i>Where might this data have been collected?</i>
Other examples	Use other varieties of Stem and Leaf Graph to discuss the data.

STATISTICS CLAIM

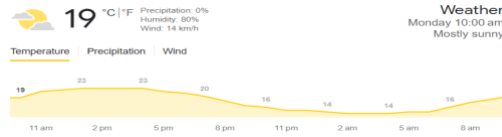
Tip Top want to know what ice cream is the most favourite in Aotearoa. They interviewed 146 people in Auckland to find out the favourite ice cream flavour of all New Zealanders.

Is that a clear representation of Aotearoa's favourite ice cream?

Big ideas	Data can vary in different ways (e.g., an object can be different sizes and colours) and it can be organised in different ways and by different characteristics (categorical, numerical). Predictions can be made through using sets of data.
Curriculum links	<ul style="list-style-type: none"> examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are believable and reasonable.
Learning Outcomes: <i>Students will be able to:</i>	<ul style="list-style-type: none"> use statistical statements to justify is a claim is believable.
Mathematical language	Statistics, data, sample, investigate, organise, sort, classify, represent, communicate, predict, outcomes.
Teacher Notes	<ul style="list-style-type: none"> The aim of this starter it to encourage students to think critically about statistical claims that are made. <p><u>Instructions</u></p> <ul style="list-style-type: none"> Present the question to the students. Give students time to reflect on the claim and then ask them to share their ideas with their partner. Notice students using statistical language in their conversations. When sharing back students ideas, encourage their peers to reason with their statements by asking do you agree or disagree with ____ idea? Other questions to pose: <ul style="list-style-type: none"> Why is it important to know the sample size? How could we make this claim more valid? What would be a good sample size to represent New Zealand? Should we believe all statistics when they are shared in the media?
Other examples	<p>Use other statements made in the media to encourage students to think critically about the maths around them.</p> <p>For example: Flu strain hits New Zealand. There has been a 30% increase in the flu virus throughout New Zealand. Questions to prompt would be: What months are they comparing the flu (summer verses winter?). Where was the sample size taken? Is it the same sample of people compared to notice the increase?</p>

PROBABILITY : TŪPONOTANGA

PROBABILITY – CONTINUUM



Ref: Google Weather (search: weather today)

The figure above shows the predicted temperature for our local area on Monday.

What is the probability it will be 23° at 4pm?

What is the probability it will be warmer at 5pm than 11am?

<p>Big ideas</p>	<p>If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes. Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.</p>
<p>Curriculum links</p>	<ul style="list-style-type: none"> • Use data visualisations to describe the distribution of observed outcomes from probability experiments and possible outcomes for theoretical probability models
<p>Learning Outcomes: <i>Students will be able to:</i></p>	<ul style="list-style-type: none"> • Compare the likelihood of events and represent these as a fraction. • Make a prediction about a chance situation.
<p>Mathematical language</p>	<p>Probability, chance, unlikely, possible, likely, certain, equal chance.</p>
<p>Teacher Notes</p>	<ul style="list-style-type: none"> • This is designed to encourage students to make predictions using mathematical language. <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Show the local weather and ask each question individually. • Encourage students to use the language of probability when explaining their reasoning. <ul style="list-style-type: none"> - Support the students to think big picture, if we know this information and can make predictions how does this help us plan... when to go swimming? When to play sport?
<p>Other examples</p>	<p>Use Google Weather to explore other regions around NZ and make probability statements.</p>

THEORETICAL PROBABILITY VERSES EXPERIMENTAL PROBABILITY

The theoretical probability is 6 is rolled on a dice is $\frac{1}{6}$.

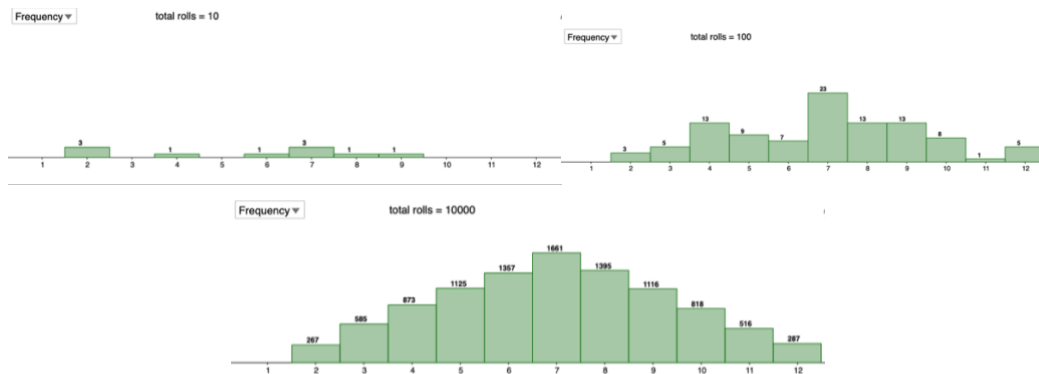
If you roll a dice 100 times and 42 of those times it lands on a six, the experimental probability is $\frac{42}{100}$.

What are the chances you would roll a six on the 101 dice roll?

Big Ideas	The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.
Curriculum Links	<ul style="list-style-type: none"> • Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain). • A probability experiment involves repeated trials. Results may vary in trials.
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Identify similarities and differences in results of trials • Compare theoretical and experimental probabilities • Make statements and form questions about trial results
Mathematical language	Trial, outcomes, sample size, theoretical probability, experimental probability, similar, different, percentage
Teacher Notes	<p>The focus of this activity is for students to understand that the results of trials will differ each time, and these results may/or may not reflect the theoretical probability.</p> <p><u>Instructions:</u></p> <ol style="list-style-type: none"> 1. Display the question and give students time to turn and talk about what they notice and what they wonder. 2. Notice students that are able to discuss the difference in theoretical probability and experimental probability. 3. Facilitate a group discussion around the idea that each dice roll is not impacted on the roll before. It is chance.
Other examples	Flip a coin 10 or 20 times and record the results and then discuss theoretical or experimental probability.

COMPARING RESULTS

These graphs show the sum of two dice after 10, 100 and 1000 trials. What do you notice? What do you wonder?



Big Ideas	The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.
Curriculum Links	<ul style="list-style-type: none"> • Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain). • A probability experiment involves repeated trials. Results may vary in trials.
Learning Outcomes <i>Students will be able to:</i>	<ul style="list-style-type: none"> • Identify similarities and differences in results of trials • Compare theoretical and experimental probabilities • Make statements and form questions about trial results
Mathematical language	Trial, outcomes, sample size, theoretical probability, experimental probability, similar, different, percentage
Teacher Notes	<p>The focus of this activity is for students to understand that the results of trials will differ each time, and these results may/or may not reflect the theoretical probability. The larger number of trials conducted, the more likely the results should reflect the theoretical probability.</p> <p><u>Instructions:</u></p> <ul style="list-style-type: none"> • Display the graphs and give students time to turn and talk about what they notice and what they wonder. • Facilitate a group discussion on what students notice and wonder. Record/annotate these ideas and ask questions that will deepen student thinking. • Prompt the students to think about if there was another 1000 trials done, what could they anticipate the graph to look like?