# RICH MATHEMATICAL TASK BOOKLET

# PROBABILITY

**YEAR 5-6** 

# **Teacher Booklet**

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Ban Heng is ordering a lunch box at a local café. He is wondering what different combinations he could order with the following options. He can choose one item from each category.

| Snack choice | Main choice      | Drink choice |
|--------------|------------------|--------------|
| Apple        | Toasted sandwich | Tea          |
| Cookie       | Sushi            | Coffee       |
|              | Pie              |              |
|              | Wrap             |              |
|              | Pizza            |              |

What are the different combinations that Ban Heng could order?

How many different combinations are there? Can you prove that you have found them all?

#### Teacher Notes

Before you launch the task, show the students the probability continuum:

Impossible Unlikely Possible Likely Certain

Ask the students to consider where they would think that 100%, 50%, and 0% chance would go on the continuum.

Ask them to think of things that they do on the weekend and put these on the continuum for themselves:

Eat some food - Go to the beach - Read a book Listen to music - See a flying pig - Play sports Make your bed - Build a snowman - Watch a movie

Ask different students to share their continuum and discuss why they might be different.

Notice whether students are able to systematically record the different options for the lunch box and work out how many different combinations are possible.

The possibility of each lunch combination could be linked to fractions in relation to the chance of each aspect and combination. This could be recorded as one-half chance for the snack, one fifth chance for each main, and one twentieth for each combination.

Expect students to use the language of probability with terms such as more likely, less likely, small chance, greater chance.

#### **Big Ideas**

When there is a chancebased situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% impossible or 100% certain).

If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes. Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

# Shareback

Select students to share that have worked systematically to find all the possible combinations.

Select students to share who have used a tree diagram or model how to organise the information like this to the students.

#### Connect

What is the probability that Ban Heng will have an apple? What is the probability that Ban Heng will have sushi? What is the probability that Ban Heng will have a pie and tea? What is the probability that Ban Heng will have a cookie, a wrap, and juice?

# Suggested Learning Outcomes

Represent the different outcomes for an event. Use a tree diagram to represent the different outcomes for an event. Find all of the possible outcomes for an event. Find the probability for an event.

# Independent Tasks

At Subway you can order different sandwich combinations. You can choose one type of bread, one filling, and one sauce.

| Bread     | Fillings  | Extra fillings |
|-----------|-----------|----------------|
| White     | Chicken   | Tomato         |
| Wholemeal | Meatballs | Lettuce        |
|           | Egg       |                |

What are the different combinations that you could order?

How many different combinations are there?

Can you prove that you have found them all?

#### Curriculum Links

During Year 5 and Year 6

Engage in chance-based investigations, including those with not equally likely outcomes, by:

- generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome

– creating data visualisations for possible outcomes

– finding probabilities as fractions

#### Mathematical Language

Probability, chance, unlikely, possible, likely, certain, equal chance.

Read the statements below and discuss whether you agree or disagree with them.

| It always rains in January                                    | Six is the hardest number to roll on $\underline{a}$ dice        |
|---|--|
| A game is fair if you follow<br>the rules.                    | You will see someone you<br>know on the way home from<br>school. |
| It is easier to get heads than<br>tails when you flip a coin. | If you buy lots of lotto tickets, you will win a prize.          |

It always rains in January

Six is the hardest number to roll on a dice

A game is fair if you follow the rules.

You will see someone you know on the way home from school.

It is easier to get heads than tails when you flip a coin.

If you buy lots of lotto tickets, you will win a prize.

Everyone in your group must agree and you should provide a range of reasons for your argument.

### **Teacher Notes**

Before the launch, ask the students to record in a tally-chart the results of a trial with blue and red jellybeans. Put the beans in a feely bag and draw them out one at a time and ask the students to record with tally-marks.

During the launch of the task, establish with the students that when Tyrone or his sister take a jellybean, it is removed from the jar and data set and not returned.

Have plastic beans and feely bags available for the students to use for the trials.

Monitor for students who are able to connect fractions with their predictions (e.g., there are 8 yellow beans which is half of the total set of 16 beans).

Notice students who are able to record and represent the results of the trials accurately using tally marks.

Expect students to use the language of probability with terms such a more likely, less likely, half chance, greater chance.

For the independent task, have beans and feely bags available for the students to use.

#### **Big Ideas**

When there is a chancebased situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ. If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

#### Curriculum Links

During Year 5 Evaluate others' statements about chancebased investigations, with justification.

During Year 6 Identify, explain, and check others' statements about chance-based investigations, referring to evidence.

# Shareback

Select students to share who indicate misconceptions such as rolling a six is more difficult so that this can be discussed.

Additionally, select students to share who qualify their statements and provide evidence for their decisions. This might include statements such as rolling a six having one sixth chance as an outcome while flipping a coin and landing on heads has a half chance as an outcome.

#### Connect

Create your own statements for other students to discuss.

# Suggested Learning Outcomes

Agree or disagree with statements related to the probability of an event. Provide reasons including quantification to support their argument. Find the probability for an event (e.g., 50% chance of heads and 50% chance of tails).

# Independent Tasks

Lani is not convinced that the chance of rolling a six on a dice is the same as other numbers.

Use the dice and undertake a trial to see if you can convince her.

First roll the dice twenty times and record the results in a tally-chart. Represent these in a column graph.

For the next trial, roll the dice 100 times and record the results in a tallychart. Represent these in a column graph.

What do you notice?

Compare your results with other students. Are they the same or different?

Combine your results with 4 other students and add up the total of rolling a dice 500 times.

What do you notice?

#### Mathematical Language

Probability, chance, unlikely, possible, likely, certain, equal chance.

The toyshop is selling marbles which come in three colours: gold, silver, and purple. They sell them in a packet with one of each colour.

The toyshop assistant had been sorting them into packets to sell, however, the larger bags spilled and got mixed up. The assistant randomly divided the marbles into bags of three to sell.

Help the toyshop assistant work out the probability of selecting three different colours of marbles for each packet if they are being selected randomly.

Begin by making a prediction for how often you think you will get one marble of each colour if you have 30 marbles (10 of each colour) and take three out randomly each time until they have all been packaged.

Now test your prediction by using the beads and feely bag. Trial this ten times and record the colours that you pull out each time.

# Teacher Notes

When you launch the task, remind students that they can represent possible outcomes using a tree diagram.

Have feely bags and beads of three different colours (10 of each and 30 in total) available for each group to experiment with the chance situation. Notice whether students are recording the results accurately and systematically and support them to do this.

Support students to make connections between probability and fractions.

The connect in this task begins to focus attention on the difference between theoretical probability and experimental probability and a comparison of this from trials. Support the students to notice that you can have different results for the same trial.

#### **Big Ideas**

When there is a chancebased situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability). A probability experiment involves repeated trials. Results can differ in different trials.

# Shareback

Select students to share that have worked out the possible ways that the marbles could be sorted when selected randomly.

For the second part of the task, choose students to share that have recorded their outcomes systematically.

#### Connect

What do you notice about your prediction and the outcome of ten trials?

Make statements about what you notice.

What is your advice for the shop assistant?

# Suggested Learning Outcomes

Identify possible outcomes in a chance situation. Compare the likelihood of events. Make a prediction about a chance situation. Collect and record data for a chance investigation.

# Independent Tasks

I choose five cubes from a feely-bag without looking. There are three red cubes and two blue cubes. What colour cubes might be in the bag and how many of each?

Give a range of solutions and explain each one.

The probability of selecting a yellow cube from a feely-bag is one in three (one third)

What colour cubes might be in the bag and how many of each?

#### Curriculum Links

During Year 5 and Year 6

Engage in chance-based investigations, including those with not equally likely outcomes, by::

– anticipating and then identifying possible outcomes for the investigative question

- generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome

During Year 6 Comparing findings from the probability experiment and associated theoretical probabilities, if the theoretical model exists

#### Mathematical Language

Chance, unfair, fair, equal chance, trial, tally chart, variation, sample, theoretical probability, experimental probability.

Mere and Mepa were discussing what happens when you roll two dice and add the total.

Mere said "If I throw the dice twenty times, I will get any total from two to twelve".

Mepa said: "No Mere, there are some totals that are more likely than others".

Who do you agree with? Why?

Test your argument using the dice and roll the dice 40 times and add the total. Record and represent the results.

## **Teacher Notes**

To launch the task, remind students of the terms theoretical probability and experimental probability by referring to the previous lesson.

Have dice available for the students to use.

Expect students to use a tree diagram or accurate form of listing to ensure that they map out the different possibilities when rolling two dice.

Notice students who are able to record and represent the results of the trials accurately using tally marks.

Expect students to use the language of probability with terms such as more likely, less likely, greater chance, theoretical probability, experimental probability.

For the independent task, have dice available.

# Shareback

Select students to share who notice and explain that particular numbers are more likely to be the sum of adding both dice by listing the possibilities and working out the probability and fraction related to this.

For the second part of the task, choose students to share that have recorded their outcomes systematically. Add the results from different students to a table and collate the results.

#### **Big Ideas**

When there is a chancebased situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

A probability experiment involves repeated trials. Results can differ in different trials.

If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

## Connect

In a game of chance, you are asked to choose one number and then roll the two dice, add the sums and see whether you have got the number. What time would you choose?

If you made the sample size larger and played the game 100 times, how many times would you expect to win?

# Suggested Learning Outcomes

Identify possible outcomes in a chance situation. Compare the likelihood of events and represent these as a fraction. Make a prediction about a chance situation. Collect and record data for a chance investigation. Compare theoretical probability with experimental probability.

# Independent Tasks

When you roll two dice would the sum of the dice be more likely to add to 9 than 10?

If you roll three dice, would the sum of the dice be more likely to add to 9 or to 10?

Use a representation or diagram to support your answer.

Now, test your prediction for three dice by rolling the dice and trialling this at least 40 times. Record and represent your results.

What do you notice?

#### Curriculum Links

During Year 5 and Year 6

Engage in chance-based investigations, including those with not equally likely outcomes, by: generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome

During Year 6 Comparing findings from the probability experiment and associated theoretical probabilities, if the theoretical model exists

#### Mathematical Language

Chance, unfair, fair, more likely, less likely, trial, tally chart, variation, theoretical probability, experimental probability, sample size.

A game uses a set of digit cards from 2 to 6 (2, 3, 4, 5, 6).

To play, you randomly pick two cards out of the bag and add the numbers together.

If the total is even then you win.

If the total is odd then you lose.

Is this a fair game?

Test your prediction using the digit card at least 40 times and add the total. Record and represent the results.

### **Teacher Notes**

Have digit cards and feely-bags available for the students to use.

Expect students to use a tree diagram or accurate form of listing to ensure that they map out the different possibilities when selecting the two digit cards.

Notice students who are able to record and represent the results of the trials accurately using tally marks.

Expect students to use the language of probability with terms such as more likely, less likely, greater chance, theoretical probability, experimental probability.

For the independent activity, provide students with the equipment needed to test their questions and also squared paper to develop graphs.

# Shareback

Select students to share who notice and explain that particular numbers are more likely to be the sum of adding both digit cards by listing the possibilities and working out the probability and fraction related to this.

For the second part of the task, choose students to share that have recorded their outcomes systematically.

#### **Big Ideas**

When there is a chancebased situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

A probability experiment involves repeated trials. Results can differ in different trials.

If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

#### Connect

Here are two more sets of digit cards. Would you choose either option to maximise your chance of winning?

5, 3, 9, 1, 4 10, 5, 4, 6, 3, 8

## Suggested Learning Outcomes

Identify possible outcomes in a chance situation. Compare the likelihood of events and represent these as a fraction. Make a prediction about a chance situation. Collect and record data for a chance investigation. Compare theoretical probability with experimental probability.

### Independent Tasks

You can work with a buddy or by yourself for this activity.

Select a question about a game from the questions that were brainstormed with your teacher at the beginning of the mathematics lesson.

Begin by considering how you will investigate the question.

What outcomes are possible in relation to your question?

What is the theoretical probability of the different outcomes? Develop a representation that shows this including a graph.

Write a plan for how you will investigate the experimental probability of the outcomes. This will need to have trials with different sample sizes. Develop representations that show these results including graphs.

Make statements about what you have found out.

Make a poster that has the following information; An introduction including what you choose to investigate. The theoretical probability and related representations. The plan to investigate the probability outcomes. The outcomes of your trials with different sample sizes and the related representations. Statements related to your findings. A conclusion.

#### Curriculum Links

During Year 5 and Year 6

Engage in chance-based investigations, by:

– anticipating and then identifying possible outcomes for the investigative question

- generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome

creating data
visualisations for possible
outcomes

- finding probabilities as fractions

 answering the investigative question

#### Mathematical Language

Chance, unfair, fair, more likely, less likely, trial, tally chart, variation, theoretical probability, experimental probability, sample size.

Tiritiri Matangi Island is an open bird sanctuary.

The Department of Conservation runs a bird banding programme to help scientists learn about bird populations. This means they catch a sample of the birds and put a band on their foot.

Today you will be helping the scientists try and work out the fraction of four species of birds and how many birds are on the island.

Firstly, decide on how you will record the data. Then take ten bird pictures from the bag and put a sticker on them to indicate a band and record the species.

Now return all the bird pictures to the bag and mix them up.

This time take 20 bird pictures from the bag and put stickers on those that are not banded. Record the species and indicate which have already been banded.

Use the samples that you have taken to predict the fraction of birds that are each species. What might the total number of birds be? How many of the total would you expect to be each species?

### **Teacher Notes**

Launch the task by asking students how they think scientists might use probability and sampling.

Explain that the problem is focused on birds and using banding and ask them to share other animals which also may be tagged. Discuss what information could be found out about birds from using banding.

Have the bird pictures, a feely-bag, and stickers for the task.

Notice students who are able to record and represent the results of the trials accurately using tally marks.

Support students to consider the sample size and accuracy but also to consider the realistic context and why it may not be possible to band all of the birds on the Island.

For the independent activity, provide students with the equipment needed to test their questions and also squared paper to develop graphs.

#### Big Ideas

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For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

# Shareback

Select students to share that have recorded their outcomes systematically and clearly indicated the number of banded birds that were chosen a second time in the larger sample of 20 birds.

#### Connect

How did your prediction for 20 game compare to the results of your trial? How did your prediction for 100 games compare to the results of the tally of the trials from the class? What do you notice?

# Suggested Learning Outcomes

How could the number of banded birds which were chosen in the second sample help us estimate the total number of birds?

How could scientists improve the reliability of their sampling?

# Independent Tasks

Select a question about a game from the questions that were brainstormed with your teacher at the beginning of the mathematics lesson.

Begin by considering how you will investigate the question.

What outcomes are possible in relation to your question?

What is the theoretical probability of the different outcomes? Develop a representation that shows this including a graph.

Write a plan for how you will investigate the experimental probability of the outcomes. This will need to have trials with different sample sizes. Develop representations that show these results including graphs.

Make statements about what you have found out.

Make a poster that has the following information; an introduction including what you choose to investigate, the theoretical probability and related representations, the plan to investigate the probability outcomes, the outcomes of your trials with different sample sizes and the related representations, statements related to your findings and a conclusion.

#### Curriculum Links

During Year 5 and Year 6

Engage in chance-based investigations by:

– anticipating and then identifying possible outcomes for the investigative question

 generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome

creating data
visualisations for possible
outcomes

– finding probabilities as fractions

- answering the investigative question

#### Mathematical Language

Chance, more likely, less likely, trial, tally chart, variation, experimental probability, sample.

Read the probability experiment poster.

What is interesting?

What is something you have learned from the poster?

What is a question that you have about the experiment?

## Teacher Notes

To launch this task, display the posters (independent activity from earlier tasks) detailing the completed probability experiments around the classroom with a piece of paper for feedback next to them.

Put the students into groups and set up a bus-stop activity where they read the poster together and then write responses to the prompts in the paper. After the timer goes off then students go to the next poster and repeat the process.

To conclude, provide students with opportunities to read the feedback about their poster and develop a reflection.

# Shareback

After the students have completed the bus-stop activity and written feedback, provide the students with time to look at the comments and feedback related to their poster.

#### Connect

Ask them to reflect and share on what they learned and what they would do differently if planning the experiment again.

# Suggested Learning Outcomes

Identify possible outcomes in a chance situation.

Make a prediction about a chance situation.

Critique and analyse the posters produced by other students detailing their probability experiments.

Ask questions about probability experiments and chance situations.

#### **Big Ideas**

When there is a chancebased situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% impossible or 100% certain).

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For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

A probability experiment involves repeated trials. Results can differ in different trials.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

# Independent Tasks

Select one or more of the following assessment tasks (attached at the end of the document) as the independent activity:

Task 1: Three coin toss

Task 2: Odd and even - three dice roll

#### Curriculum Links

During Year 5 and Year 6

Engage in chance-based investigations by:

- anticipating and then identifying possible outcomes for the investigative question

- generating all possible ways to get each outcome (a theoretical approach), or undertaking a probability experiment and recording the occurrences of each outcome

– creating data visualisations for possible outcomes

- finding probabilities as fractions

- answering the investigative question

#### Mathematical Language

Chance, more likely, less likely, trial, tally chart, variation, experimental probability, sample size, theoretical probability.

# Assessment Task 1 - Probability - Year 5-6

What happens when you flip three coins? Show all the possible outcomes and predict what is most likely and least likely.

Now test your predictions by flipping three coins. Choose how many times you will trial the coin flip. Record and represent your results. What do you notice? Make statements.

#### Assessment Task 2 - Probability - Year 5-6

In odds and evens, you roll the dice and record whether it is odd (1, 3, 5) or even (2, 4, 6). What happens when you roll the three dice at the same time? Show all the possible outcomes and predict what is most likely and least likely.

Now test your predictions by rolling the dice and recording the combinations of odd and even numbers. Choose how many times you will trial the dice roll. Record and represent your results. What do you notice? Make statements.