RICH MATHEMATICAL TASK BOOKLET

PROBABILITY

YEAR 7-8

Teacher Booklet

Task 1

At the Melbourne Cup, there is a 'trifecta' in a six-horse race. Each choice costs $1.

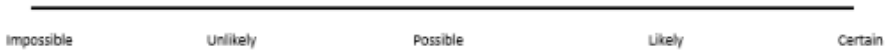
To win the 'trifecta', you need to pick the first three horses in the order they finish.

How much would it cost you to cover the field?

If the trifecta pays $83, is it a good idea to cover the field?

Teacher Notes

Before you launch the task, show the students the probability continuum:



Ask the students to consider where they would think that 25% and 75% chance would go on the continuum.

Ask them to think of the weather today and arrange the weather descriptions on the continuum:
Windy Sunny Rain Snow
Cloudy Lightening

Notice whether students are able to systematically record the different options for the order of the horses and work out how many different combinations are possible. It is important for students to realise that the order matters for the horses.

The possibility of each combination of order of horses could be linked to fractions in relation to the chance of each combination.

Expect students to use the language of probability with terms such as more likely, less likely, small chance, greater chance and connect this to the probability continuum and percentages.

Shareback

Select students to share that have worked systematically to find all the possible combinations.

Select students to share who have used a tree diagram to organise the information.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

Connect

If there was four horses racing, would covering the field be a good option for a pay-out of $83?

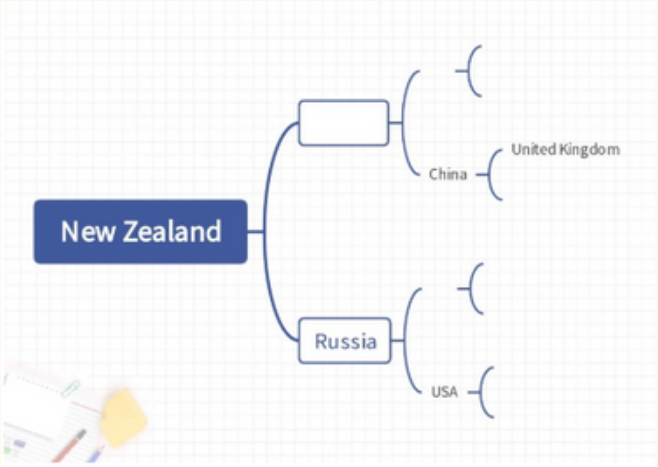
Suggested Learning Outcomes

Represent the different outcomes for an event.
Use a tree diagram to represent the different outcomes for an event.
Find all of the possible outcomes for an event.
Find the probability for an event.

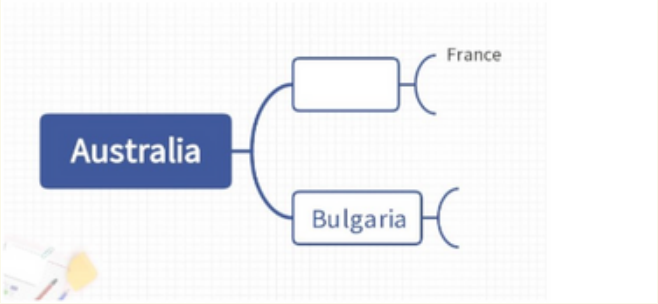
Independent Tasks

This diagram shows the draws for the World Chess Tournament. All the countries that lost their first round championship matches went into the consolation playoff.

First round:



Play off



Recreate the tree diagrams and fill all the entries of the draw.

Curriculum Links

During Year 7 ***and Year 8***

Conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:

- identifying and systematically listing possible answers to the investigative question

– collecting and recording data

– creating data visualisations for the distribution of observed outcomes ***and for all possible outcomes for theoretical probability models where they exist***

- describing what these visualisations show

- finding the probability estimates for the different outcomes

- answering the investigative question

- reflecting on anticipated outcomes

Mathematical Language

Probability, chance, unlikely, possible, likely, certain, equal chance.

Anticipations

Solutions, Misconceptions

Task 2

Zhao is playing a game with his sister. They put 10 beans in a bag with six blue beans and four yellow beans They choose one out and then replace it.

If you did this four times, what is the probability that you would pick at least one yellow bean?

What happens if you don't replace the bean each time?

Use the beans and a bag and test the game both ways by trialling picking out the beans for differing numbers of trials. Record your results and make statements about what you notice.

Teacher Notes

During the launch remind students how to record outcomes using a tally chart.

Have beans and bags available.

Notice whether students are recording the results accurately and systematically and support them to do this.

Support students to make connections between probability and fractions. Facilitate them to represent the chance of as both a fraction and a percentage.

The connect in this task focuses attention on the difference between theoretical probability and experimental probability and a comparison of this from trials. Support the students to notice that you can have different results for the same trial. Also, that the more trials (the greater the sample size) the more likely the distribution of the experimental probability will be closer to the theoretical probability.

For the independent activity, have coins available to toss and squared paper to develop graphs.

Shareback

Select students to share that have worked out the probability for both scenarios and can connect this to fractions.

For the second part of the task, choose students to share that have recorded their outcomes systematically.

Add the results from different students to a table of possible outcomes and collate the results for both scenarios.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

A probability experiment involves repeated trials. Results can differ in different trials.

If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

Connect

What do you notice about your prediction (introduce term theoretical probability) and the outcome of ten trials (introduce term experimental probability)?

What do you notice about the outcome when the trials are added together and the sample size is extended?

What does this tell you about probability.

Suggested Learning Outcomes

Identify possible outcomes in a chance situation.
Compare the likelihood of events and represent these as a fraction.
Make a prediction about a chance situation.
Collect and record data for a chance investigation.
Compare theoretical probability with experimental probability.
Recognise theoretical probability is influenced by a smaller sample of trials.

Independent Tasks

At the school fair, one of the stalls has a coin toss. To win you have to predict the outcome of tossing four coins. If your coin toss matches the prediction, you win a prize.

What are the possible outcomes? What is the chance of getting each outcome and likelihood of winning a prize?

Use the coin and test the game by trialling tossing the coins for ten trials. Record your results in the table and represent them on a column graph.

Now test the game by trialling tossing the coins 40 more times and record the results and total the outcomes.

Make statements about what you notice.

Curriculum Links

During Year 7 ***and Year 8***

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identifying and systematically listing possible answers to the investigative question

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– creating data visualisations for the distribution of observed outcomes ***and for all possible outcomes for theoretical probability models where they exist***

- describing what these visualisations show

- finding the probability estimates for the different outcomes

- answering the investigative question

- reflecting on anticipated outcomes

Mathematical Language

Chance, unfair, fair, equal chance, trial, tally chart, variation, theoretical probability, experimental probability, sample size.

Anticipations

Solutions, Misconceptions

Task 3

Isla and Annabelle are playing Mastermind. To play the game you have multiple pegs of three different colours (green, yellow, and blue).

Player One chooses two pegs and puts them into the pegboard behind a screen.

Player Two has the guess the colour of the two pegs and their position on the board in front of the screen.

What is the minimum number of turns you would need to be sure you have it right?

Each time Player Two has a turn then Player One gives them feedback in the following way:
If the peg is the right colour but in the wrong place, they give you one point.
If the peg is the right colour and right place, they give you two points.

Player Two tells you how many points that you have.
If you are given two points, what could this mean?

With the feedback, what would be the minimum number of turns to make sure you have it right?

Teacher Notes

Launch the task by asking students to consider the different combinations for the pegs.

Expect students to use a tree diagram or accurate form of listing to ensure that they map out the different possibilities.

Have peg boards or counters available for the students to use and model the solutions.

Facilitate students to notice that two points could mean two possibilities, either one peg is right colour and right place, or both pegs the right colour but wrong places.

Encourage the students to discuss the different points that could be allocated.
Notice students who are able to generate specific strategies that increase the probability of a correct turn (e.g., using two of the same colour pegs).
Expect students to use the language of probability with terms such as more likely, less likely, greater chance.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

Curriculum Links

During Year 7 ***and Year 8***

Conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:

- anticipating what outcomes are possible

- identifying and systematically listing possible answers to the investigative question

- collecting and recording data

Shareback

Select students to share who notice that there are nine different and possibilities for the peg design and the related fraction.

For the second part of the task, choose students to share who have recorded different options systematically and worked out a strategy to show that it is possible to work out the solution with feedback in four turns.

Connect

What are the best strategies for getting the correct answer in the least number of moves?

Suggested Learning Outcomes

Identify possible outcomes in a chance situation.
Compare the likelihood of events and represent these as a fraction or a percentage.
Make a prediction about a chance situation.

Independent Tasks

If you flip a coin three times in a row, what is the probability that you will get a tail on at least one of the flips?

Draw a tree diagram to help you work this out.

What is the probability of getting no tails?

What would be a quick way to work out the probability of getting at least one tail if you flip the coin 4 times or 5 times or more?

Can you come up with a rule that you could use to work this out?
There is a class with 13 girls and 11 boys. Three children are chosen at random to help the teacher.

What is the probability there will be at least one boy chosen?

Curriculum Links

- creating data visualisations for the distribution of observed outcomes ***and for all possible outcomes for theoretical probability models where they exist***

- describing what these visualisations show

- finding the probability estimates for the different outcomes

- proposing possible theoretical outcomes and associated probabilities, for situations where no theoretical model exists

- answering the investigative question

- identifying similarities and differences between their findings and those of others

- reflecting on anticipated outcomes

- comparing findings from the probability experiment and associated theoretical probabilities, as appropriate

- identifying similarities and differences between findings from the probability experiment and associated theoretical probabilities, as appropriate

Mathematical Language

Chance, unfair, fair, more likely, less likely, variation, theoretical probability.

Anticipations

Solutions, Misconceptions

Task 4

The Department of Conservation runs a bird banding programme to help scientists learn about bird populations. This means they catch a sample of the birds and put a band on their foot.

Today you will be helping the scientists try and work out the fraction of six species of birds and how many birds are in a bird sanctuary area in the South Island.

Firstly, decide on how you will record the data. Then take 15 bird pictures from the bag and put a sticker on them to indicate a band and record the species.

Now return all the bird pictures to the bag and mix them up.

This time take 30 bird pictures from the bag and put stickers on those that are not banded. Record the species and indicate which have already been banded.

Use the samples that you have taken to predict the fraction of birds that are each species. What might the total number of birds be? How many of the total would you expect to be each species?

Teacher Notes

Before launching this task, ask students to brain-storm games that they play that involve chance. This could include games such as Yahtzee, Monopoly, Game of Life, card games.

Ask them to brainstorm questions related to the games that they could investigate, this could include the likelihood of different outcomes in Yahtzee, the probability of rolling doubles in monopoly, the spinner landing on specific sets of numbers for Game of Life or the likelihood of turning over particular cards.

Record the questions that could be investigated on the board for the students to select from. This will be the basis of the independent activity and a probability investigation. Let students know that they can either work by themselves or with a partner for the investigation.

Launch the task by asking students how they think scientists might use probability and sampling.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

Teacher Notes - continued

Explain that the problem is focused on birds and using banding and ask them to share other animals which also may be tagged. Discuss what information could be found out about birds from using banding.

Have the bird pictures, a feely-bag, and stickers for the task.

Notice students who are able to record and represent the results of the trials accurately using tally marks.

Support students to consider the sample size and accuracy but also to consider the realistic context and why it may not be possible to band all of the birds on the island.

For the independent activity, provide students with the equipment needed to test their questions and also squared paper to develop graphs.

Shareback

Select students to share that have recorded their outcomes systematically and clearly indicated the number of banded birds that were chosen a second time in the larger sample of 30 birds.

Connect

How could the number of banded birds which were chosen in the second sample help us estimate the total number of birds?

How could scientists improve the reliability of their sampling?

Suggested Learning Outcomes

Identify possible outcomes in a chance situation.
Make a prediction about a chance situation.
Collect and record data for a chance investigation.

Curriculum Links

During Year 7 ***and Year 8***

Conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:

- identifying and systematically listing possible answers to the investigative question

- collecting and recording data

- creating data visualisations for the distribution of observed outcomes

- describing what these visualisations show

- finding the probability estimates for the different outcomes

- answering the investigative question

- proposing possible theoretical outcomes and associated probabilities, for situations where no theoretical model exists

- reflecting on anticipated outcomes

- comparing findings from the probability experiment and associated theoretical probabilities, as appropriate

Mathematical Language

Chance, more likely, less likely, trial, tally chart, variation, experimental probability, sample.

Independent Tasks

You can work with a buddy or by yourself for this activity.

Select a question about a game from the questions that were brainstormed with your teacher at the beginning of the mathematics lesson.

Begin by considering how you will investigate the question.

What outcomes are possible in relation to your question?

What is the theoretical probability of the different outcomes?
Develop a representation that shows this including a graph.

Write a plan for how you will investigate the experimental probability of the outcomes. This will need to have trials with different sample sizes. Develop representations that show these results including graphs.

Make statements about what you have found out.

Make a poster that has the following information;
An introduction including what you choose to investigate.
The theoretical probability and related representations.
The plan to investigate the probability outcomes.
The outcomes of your trials with different sample sizes and the related representations.
Statements related to your findings.
A conclusion.

Anticipations

Solutions, Misconceptions

Task 5

Lola has some unusual dice and has invented a game with these.

There is a white dice with the numbers: 1, 1, 6, 6, 8, 8 on each face.

There is a yellow dice with the numbers: 2, 2, 4, 4, 9, 9 on each face.

There is a green dice with the numbers; 3, 3, 5, 5, 7, 7.

To play you each choose one of the dice and roll it at the same time. The player with the biggest number wins.

Does it make a difference who gets to choose the dice first?

Which dice should you select dependent on what the first player chooses?

Teacher Notes

Before you launch the task, have the non-transitive dice available and let students play the game and experiment with these. Explain that these dice use non-consecutive numbers.

Have cubes available to create the dice.

Expect students to use a tree diagram or accurate form of listing to ensure that they map out the different possibilities for the dice.

Expect students to use the language of probability with terms such as more likely, less likely, greater chance, theoretical probability, experimental probability.

Shareback

Select students to share who notice and explain that the first dice chosen has an impact on the second choice. This can be shown through diagrams and by using a tree diagram to list all possibilities.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

A probability experiment involves repeated trials. Results can differ in different trials.
If all possible outcomes in a chance situation are equally likely, the probability of an event happening is a fraction where the numerator is the number of ways the event can happen, the denominator is the total number of possible outcomes.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

Connect

Develop a set of three dice yourself that are non-transitive, explain what the best choices for two players would be for your dice depending on which dice is chosen first.

Suggested Learning Outcomes

Identify possible outcomes in a chance situation.
Compare the likelihood of events and represent these as a fraction.
Make a prediction about a chance situation.

Independent Tasks

You can work with a buddy or by yourself for this activity.

Select a question about a game from the questions that were brainstormed with your teacher at the beginning of the mathematics lesson.

Begin by considering how you will investigate the question.

What outcomes are possible in relation to your question?

What is the theoretical probability of the different outcomes?
Develop a representation that shows this including a graph.

Write a plan for how you will investigate the experimental probability of the outcomes. This will need to have trials with different sample sizes. Develop representations that show these results including graphs.

Make statements about what you have found out.

Make a poster that has the following information;
An introduction including what you choose to investigate.
The theoretical probability and related representations.
The plan to investigate the probability outcomes.
The outcomes of your trials with different sample sizes and the related representations.
Statements related to your findings.
A conclusion.

Curriculum Links

During Year 7 ***and Year 8***

Conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:
- creating data visualisations for the distribution of observed outcomes ***and for all possible outcomes for theoretical probability models where they exist***
- finding the probability estimates for the different outcomes

Identify, explain, and check others' statements about chance-based investigations, referring to evidence

Mathematical Language

Chance, unfair, fair, more likely, less likely, trial, tally chart, variation, theoretical probability, experimental probability, non-transitive dice.

Anticipations

Solutions, Misconceptions

Task 6

Read the statements below and discuss whether you agree or disagree with them.

In Lotto it is very likely that the balls selected will be a mixture of tens numbers. You are more likely to win if you select numbers from a mixture of tens numbers.	The weather is completely random. It's so mixed, and even if the forecasters try very hard, they don't always get it right.
No-one can work out how tall you will be when you grow up, you have to wait and see. This means it is random.	You make a dice with the numbers 4, 4, 4, 4, 4, 6. If you roll it a lot and write down the numbers, they are random numbers.
The weather tomorrow isn't random, it depends on the weather today.	If you flip a coin 10 times and get heads every time, then something must be wrong with the coin or how you are flipping it.
In Lotto, you should not pick consecutive numbers because it would be almost impossible for them to come up.	If you do 5 divided by 19 on a calculator the answer is 0.263157894… this means the decimal digits are random.

Everyone in your group must agree and you should provide a range of reasons for your argument

Teacher Notes

Before the launch, ask students to brainstorm and share their initial ideas of randomness and what this might mean in relation to mathematics. Record the ideas on the board.

During the launch, ask the students to come up with a sentence that defines randomness. Share the definitions and ask the other students to agree or disagree. Record those that are agreed with on the board.

Monitor for students who may indicate misconceptions about probability and randomness, for example, stating if you toss a coin and get a head lots of times then it means the coin is faulty.

Notice students who qualify their statements further by referring to definitions of randomness to support their arguments.

Expect students to use the language of probability in their discussion and to provide thoughtful arguments for their responses to the statements.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

Randomness involves uncertainty. It means that there is no pattern or possibility of predicting what will happen.

Curriculum Links

During Year 7 ***and Year 8***

Plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:

- posing an investigative question

- anticipating what outcomes are possible

- identifying and systematically listing possible answers to the investigative question

- collecting and recording data

Shareback

Select students to share who have different answers in regards to the item. Support the students to engage in mathematical argumentation and to qualify their statements and provide reasons based on the initial definitions of randomness.

Connect

What are the key ideas and a working definition of randomness in probability?

Suggested Learning Outcomes

Agree or disagree with statements related to the randomness of an outcome. Provide reasons to support their argument about randomness or non-randomness.

Independent Tasks

Finish developing your chance investigation and poster so it is ready to present tomorrow.

Make a poster that has the following information;
An introduction including what you choose to investigate.
The theoretical probability and related representations.
The plan to investigate the probability outcomes.
The outcomes of your trials with different sample sizes and the related representations.
Statements related to your findings.
A conclusion.

Curriculum Links

During Year 7 ***and Year 8***

- creating data visualisations for the distribution of observed outcomes ***and for all possible outcomes for theoretical probability models where they exist***

- describing what these visualisations show

- finding the probability estimates for the different outcomes

- proposing possible theoretical outcomes and associated probabilities, for situations where no theoretical model exists

- answering the investigative question

- identifying similarities and differences between their findings and those of others

- reflecting on anticipated outcomes

- comparing findings from the probability experiment and associated theoretical probabilities, as appropriate

- identifying similarities and differences between findings from the probability experiment and associated theoretical probabilities, as appropriate

Mathematical Language

Probability, chance, unlikely, possible, likely, certain, random, independent event.

Anticipations

Solutions, Misconceptions

Task 7

Read your classmates' probability experiment posters.
Make notes on each poster about:

What is interesting?
What is something you have learned from the poster?
What is a question that you have about their experiment?

Teacher Notes

To launch this task, display the posters (independent activity from earlier tasks) detailing the completed probability experiments around the classroom with a piece of paper for feedback next to them.

Put the students into groups and set up a bus-stop activity where they read the poster together and then write responses to the prompts in the paper. After the timer goes off then students go to the next poster and repeat the process.

To conclude, provide students with opportunities to read the feedback about their poster and develop a reflection.

Shareback

After the students have completed the bus-stop activity and written feedback, provide the students with time to look at the comments and feedback related to their poster.

Connect

Ask them to reflect and share on what they learned and what they would do differently if planning the experiment again.

Suggested Learning Outcomes

Identify possible outcomes in a chance situation.
Make a prediction about a chance situation.
Critique and analyse the posters produced by other students detailing their probability experiments.
Ask questions about probability experiments and chance situations.

Big Ideas

When there is a chance-based situation, there are sets of possible outcomes that can be arranged into events. Probability is the chance of an event occurring. This can be represented with language or values (e.g., 0% - impossible or 100% certain).

For some situations or games, the chance or probability of particular outcomes can be calculated (theoretical probability). Theoretical probability and what happens in an experiment will differ.

For some situations or games using repeated testing can give a sense of which outcomes are more likely (experimental probability).

A probability experiment involves repeated trials. Results can differ in different trials.

Data visualisations can be used to show what outcomes are possible and more likely. They can also be used to represent the results of a probability investigation.

Curriculum Links

During Year 7 ***and Year 8***

Conduct probability experiments for chance-based situations, including undertaking a large number of trials using digital tools, by:

- posing an investigative question

Independent Tasks

Select one or more of the following assessment tasks (attached at the end of the document) as the independent activity:

Task 1: Digit cards

Task 2: Non-transitive dice

Curriculum Links continued

- anticipating what outcomes are possible and which of them are more or less likely to occur

- identifying and systematically listing possible answers to the investigative question

- collecting and recording data

- creating data visualisations for the distribution of observed outcomes ***and for all possible outcomes for theoretical probability models, where they exist***

- finding the probability estimates for the different outcomes

- proposing possible theoretical outcomes and associated probabilities, for situations where no theoretical model exists

- answering the investigative question

-identifying similarities and differences between their findings and those of others

- reflecting on anticipated outcomes

Identify, explain, and check others' statements about chance-based investigations, referring to evidence.

Mathematical Language

Chance, more likely, less likely, trial, tally chart, variation, experimental probability, sample size, theoretical probability.

Anticipations

Solutions, Misconceptions

Assessment Task 1 - Probability - Year 7 & 8

John invents a game with four-digit cards numbered 1 to 4. To play you choose two of the cards and multiply the numbers together. If you get an odd number, you win a point. If you get an even number then John wins a point.
Is the game fair? Show all the possible outcomes and the chance of each.

Now test your predictions by using the digit cards and drawing out two each time. Choose how many times you will trial game. Record and represent your results. What do you notice? Make statements.

Assessment Task 2 - Probability - Year 7 & 8

Tia invents a game with some dice that she has made. It has 6 faces which she has labelled 1, 2, 3, 5, 7, 9. To play you roll both dice and add the numbers together. If the sum is an odd number, you win a point. If the sum is an even number then Tia wins a point.
Is the game fair? Show all the possible outcomes and the chance of each.

Now test your predictions by using the dice and rolling both each time. Choose how many times you will trial game. Record and represent your results. What do you notice? Make statements.