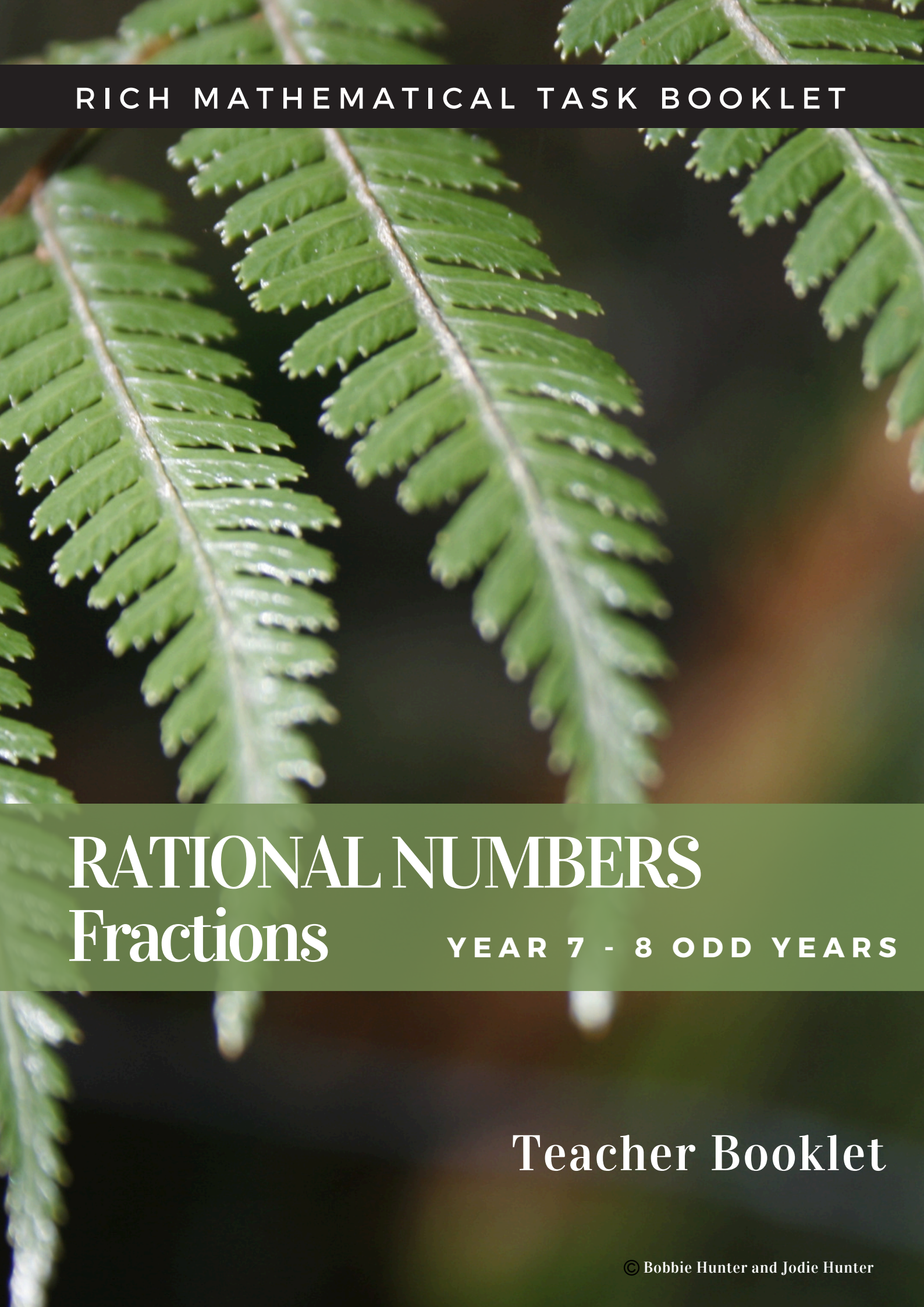

RICH MATHEMATICAL TASK BOOKLET

RATIONAL NUMBERS
Fractions

YEAR 7 - 8 ODD YEARS

Teacher Booklet

Task 1

What are all the different ways you can use the fraction tiles to make more than one quarter but less than seven ninths?
As you make these record them and be ready to explain and justify why they are less than one whole.

What are all the different ways you can use the fraction tiles to make a fraction number that is less than four eighths but more than one seventh?
As you make these record them and be ready to explain and justify why they are less than one half.

What are all the different ways you can use the fraction tiles to make a fraction number that is more than one whole but less than one whole and two thirds?
As you make these record them and be ready to explain and justify why they are more than one whole.

What are all the different ways you can use the fraction tiles to make a fraction number that is the same as four fifths?
As you make these record them and be ready to explain and justify why they are equivalent.

What are all the different ways you can use the fraction tiles to make a fraction number that is the same as two sevenths?
As you make these record them and be ready to explain and justify why they are equivalent.

Teacher Notes

Before the launch have the students explore and talk together about when they have used fractions outside of school.

Have fraction tiles available for the students to use to prove and justify.

Monitor for students using the words fractional numbers (not pieces or bits) and greater than, less than, the same as.

Notice students who make conjectures during connect and have students explore and prove these ideas.

Expect students to represent materials and use appropriate notation and the equal sign ($\frac{3}{4} = \frac{6}{8}$).

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Shareback

Select students to share who made combinations of the whole or other fractions using the same size pieces (halves, quarters, sixths, eighths) and recorded the combinations as numbers, equations, or words. Then, select students to share who made combinations using the unlike fraction pieces (e.g., $\frac{1}{4}+\frac{2}{9}$ is more than a quarter and less than $\frac{7}{9}$) and recorded the combinations as numbers, equations or words.

Connect

Generate sets of fractions that are the same as $\frac{1}{4}$.

Can you notice any patterns or rules for why fractions are the same as $\frac{1}{4}$?

Generate sets of fractions that are the same as $\frac{4}{5}$.

Can you notice any patterns or rules for how you can find fractions that are the same as $\frac{4}{5}$?

Generate sets of fractions that are the same as $\frac{2}{3}$.
Can you notice any patterns or rules for how you can find fractions that are the same as $\frac{2}{3}$?

Suggested Learning Outcomes

Combine and recombine different units of fractions to make one whole.

Identify and make equivalent fractions.

Recognise and use improper fractions to represent more than one whole.

Independent Tasks

1. $\frac{1}{6}$ of 36 =
2. $\frac{5}{6}$ of 36 =
3. $\frac{1}{2}$ of 200 =
4. $\frac{2}{5}$ of 100 =
5. $\frac{3}{5}$ of 200 =
6. $\frac{1}{4}$ of 280 =
7. $\frac{2}{9}$ of 540 =
8. $\frac{3}{4}$ of 1 000 =
9. $\frac{1}{3}$ of ? = 24
10. $\frac{3}{9}$ of ? = 27

Curriculum Links

During Year Seven and Eight

identify, read, write, and represent fractions, decimals, and percentages

compare, order, and convert between fractions, decimals, and percentages

find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers

Mathematical Language

Whole, half, halves, quarters, fourths, thirds, sixths, ninths, twelfths, eighths, whole number, equal, equivalent, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 2

This question is asked on a quiz show:

Are there numbers between numbers?

What would you answer be and why? Prove your answer by recording the following:

5 numbers that might come before 1
15 numbers that might come between 5 and 6

For the doubters in the room be ready to use a representation and a number line to prove that there are the numbers you have recorded and a whole lot more.

Teacher Notes

Facilitate the students to notice that earlier in the year, the number lines they have used only contained whole numbers (numbers that resulted from counting). The fractions they are talking about now (numbers resulting from equal splitting or partitioning) can be represented on the number line. This shows that fractions may also be thought of as numbers. In the connection refer to the fractions (e.g., $3\frac{1}{2}$ as a mixed number.

Monitor for students using vocabulary which emphasises dividing or splitting equally of portioning into equal parts.

Notice students who find the concept of fractions as numbers between numbers counter intuitive. Allow them to struggle and construct reasoning through mathematical talk and using agreeing mathematically and disagreeing mathematically (e.g., I agree because...).

Shareback

Select students to share who have developed a representation and marked the fraction on a number-line. Draw a number-line on the board and ask all students who share to mark the fraction onto it by partitioning the number-line.

Big Ideas

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A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Each fraction can be associated with a unique point on a number line.

There is no least or greatest fraction on the number line.

There are an infinite number of fractions between any two fractions on the number line

Connect

What mixed numbers are there between 5 and 10?

What would the number be as an improper fraction?

Mark each one on a number-line to show where they would be.

Suggested Learning Outcomes

Record fractions on a number-line.

Find and associate fractions with a unique point on a number line.

Recognise there are an infinite number of fractions between any two whole numbers or any two fractions on a number line.

Independent Tasks

Write a set of 10 quiz questions using all the fraction knowledge you have. Make sure that you record the answer on the back of each quiz question.

Curriculum Links

During Year Seven and Eight

identify, read, write, and represent fractions, decimals, and percentages

compare, order, and convert between fractions, decimals, and percentages

find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers

Mathematical Language

Whole, half, halves, quarters, fourths, thirds, sixths, twelfths, eighths, fraction, whole number, equal, equivalent, mixed numbers, equivalent, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 3

John and Henry are having an argument over their homework. They were asked to order a set of fractions from smallest to largest.

John has ordered the fractions in this order:

$\frac{3}{4}$ $\frac{4}{5}$ $\frac{7}{10}$ $\frac{10}{12}$ $\frac{12}{15}$

Henry disagrees with John. To prove his claim, Henry explains to John a different way to order the fractions. Use representations including a number line to show how you would order the fractions from smallest to largest.

Teacher Notes

Facilitate the students to notice that the denominator represents the number of pieces the whole has been divided into and the numerator represents the number of pieces.

Expect students to represent using a range of representations including the use of a number line or rectangular blocks which are the same size.

In the connect, students may generalise that the larger the denominator the smaller the fraction piece left is, however, facilitate them to notice that this only applies to unit fractions and is not a rule for all fractions.

Shareback

Select students to share who converted fractions to equivalent fractions using informal methods with representations before they compared the fractions. If any students changed all the fractions to a common denominator than select them to share last.

If no students use the strategy of findings the lowest common denominator, then introduce this as a strategy to the class.

Big Ideas

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On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Connect

Put these in order from smallest to largest by finding the lowest common denominator

$\frac{3}{4}$ $\frac{6}{7}$ $\frac{12}{16}$

Does this always work?

Support students to notate:

4, 8, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56

7, 14, 21, 28, 35, 42, 49, 56

14, 28, 42, 56

Ensure that the students generalise the process of finding the highest.

Suggested Learning Outcomes

Order and compare fractions.

Find equivalent fractions.

Independent Tasks

Place these fractions on a numberline in order of size

1. $\frac{5}{6}, \frac{2}{3}, \frac{7}{9}$
2. $\frac{3}{4}, \frac{5}{8}, \frac{7}{12}$
3. $\frac{7}{8}, \frac{11}{12}, \frac{10}{11}$
4. $\frac{3}{8}, \frac{5}{7}, \frac{1}{2}$
5. $\frac{5}{6}, \frac{8}{12}, \frac{7}{9}$
6. $\frac{11}{16}, \frac{5}{8}, \frac{7}{12}$
7. $\frac{7}{8}, \frac{2}{3}, \frac{3}{4}$
8. $\frac{1}{4}, \frac{3}{8}, \frac{1}{3}$

For each set of fractions what rule or pattern did you use to help you?

Curriculum Links

During Year Seven and Eight

identify, read, write, and represent fractions, decimals, and percentages

compare, order, and convert between fractions, decimals, and percentages

find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers

Mathematical Language

Whole, quarters, fourths, thirds, sixths, twelfths, eighths, ninths, fifteenths, fraction, equal, equivalent, mixed numbers, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 4

Amataga's family have been making an Ie tōga to present at the opening of their new church.
By last month they had completed three eighths of it.
Last week was busy so they only completed another third of it. This week they have completed another quarter.
How much have they completed and how much more do they have to do to complete it?

Teacher Notes

An Ie toga is a Samoan fine mat which is presented on ceremonial occasions and highly treasured. During the launch ensure that the students recognise these as of great cultural importance (never used as floor mats). Establish the maths norms using the concept of how a group of people work together as a family to make a fine mat.

Facilitate students to notice that to add fractions the denominators need to be the same.

Monitor for students using vocabulary of equivalence and relational thinking.

Notice students who show relational understanding ($\frac{3}{8}$ as $\frac{1}{8}+\frac{1}{4}$ or $\frac{6}{16}$).

In the connect develop a generalisation for rules to find a common denominator through finding a common multiple.

Shareback

Select students to share who converted fractions to equivalent fractions using informal or more formalised methods (multiplication for example) before they added the fractions.

Big Ideas

Numbers can be described in many different ways including as fractions.

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Connect

What are the common denominators for:

$\frac{1}{4}$ and $\frac{1}{8}$

$\frac{1}{5}$ and $\frac{1}{10}$

$\frac{1}{2}$ and $\frac{1}{6}$ and $\frac{1}{8}$

$\frac{1}{9}$ and $\frac{1}{4}$ and $\frac{2}{3}$

$\frac{5}{7}$ and $\frac{2}{6}$ and $\frac{1}{3}$

What do you notice about finding common denominator?

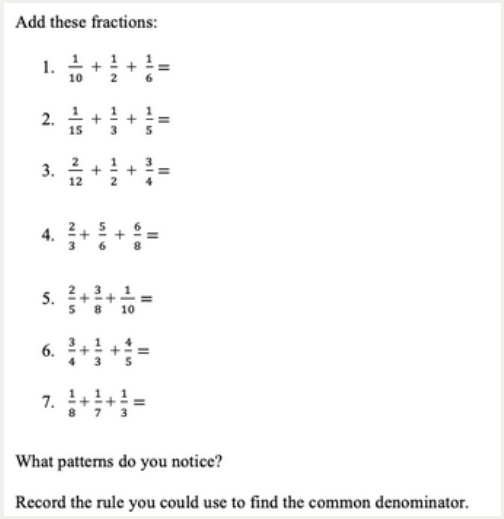
Can you describe a rule to use when adding fractions with different denominators?

Suggested Learning Outcomes

Change fractions to equivalent fractions

Solve problems that involve adding or subtracting fractions.

Independent Tasks



Add these fractions:

1. $\frac{1}{10} + \frac{1}{2} + \frac{1}{6} =$
2. $\frac{1}{15} + \frac{1}{3} + \frac{1}{5} =$
3. $\frac{2}{12} + \frac{1}{2} + \frac{3}{4} =$
4. $\frac{2}{3} + \frac{5}{6} + \frac{6}{8} =$
5. $\frac{2}{5} + \frac{3}{8} + \frac{1}{10} =$
6. $\frac{3}{4} + \frac{1}{3} + \frac{4}{5} =$
7. $\frac{1}{8} + \frac{1}{7} + \frac{1}{3} =$

What patterns do you notice?

Record the rule you could use to find the common denominator.

Curriculum Links

During Year Seven and Eight

add and subtract fractions with different denominators, using equivalent fractions

Mathematical Language

Whole, quarters, fourths, thirds, sixths, twelfths, eighths, ninths, fifteenths, fraction, equal, equivalent, mixed numbers, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 5

How good are your estimating skills? Which is the answer closest to?

$\frac{1}{12} + \frac{7}{8} =$

Is it 20, 8, $\frac{1}{2}$, 1?

$\frac{8}{9} + \frac{11}{12} =$

Is it 19, $1\frac{1}{2}$, 2, $1\frac{3}{4}$?

$\frac{6}{7} - \frac{1}{2} =$

Is it $\frac{1}{2}$, 1, 5

$9\frac{3}{4} - 5\frac{5}{6} =$

Is it 4, $3\frac{1}{2}$, 3?

Be ready to justify your reasoning using at least two different ways.

Teacher Notes

Reinforce to the students that estimation is a thinking tool and it should always be used to make sense of the size of the number.

Notice students who reason out the size of the fractions rather than working through a procedure to get the exact answer.

Shareback

Select students to share who have first estimated of the size of the fractions in the equation and then estimated the size of the overall answer rather than giving the exact answer.

Big Ideas

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Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Connect

Which are greater than one or less than one?

$\frac{1}{4} + \frac{1}{4} + \frac{1}{4}$

$\frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8} + \frac{2}{8}$

$\frac{1}{2} + \frac{3}{4}$

$\frac{19}{20} + \frac{1}{10}$

$\frac{1}{100} + \frac{1}{20}$

What do you noitice? Be ready to explain and justify your reasoining.

Suggested Learning Outcomes

Change fractions to equivalent fractions.

Solve problems that involve adding fractions.

Independent Tasks

Solve these equations using estimation as your thinking tool. Provide justification for each answer without working out the exact answer.
Record whether they are more or less than a $\frac{1}{2}$?

1. $\frac{2}{5}+\frac{1}{10}$
2. $\frac{1}{4}+\frac{3}{8}$
3. $\frac{1}{3}+\frac{1}{6}$
4. $\frac{3}{7}+\frac{1}{4}$

Record whether they are more or less than 2?

5. $\frac{3}{4}+\frac{9}{12}+\frac{1}{2}$
6. $\frac{17}{20}+\frac{7}{8}+\frac{9}{10}$
7. $3\frac{1}{6}-1$
8. $3\frac{3}{4}-1\frac{1}{2}$

Now your turn to write a set of ten questions for your classmates to answer similar to the ones above.

Curriculum Links

During Year Seven and Eight

add and subtract fractions with different denominators, using equivalent fractions

Mathematical Language

Whole, quarters, fourths, thirds, sixths, equal, equivalent, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 6

A car travels $\frac{5}{6}$ of a kilometre in one minute. How many kilometres will the car have travelled after 11 minutes?

A recipe to make 12 cookies needs $\frac{2}{5}$ of a cup of sugar. Litea wants to make enough cookies for her hub at school which has 84 students. How much sugar does Litea need?

Timo is preparing bags of mandarins to sell at the local market. Each bag has $\frac{3}{4}$ of a kilogram of mandarins. Timo needs to prepare 15 bags to sell, how many kilograms of mandarins does he need to fill the bags?

Teacher Notes

Facilitate the students to notice that there are multiples of the fractional number which they can add or multiply

Notice students who use multiplicative thinking. Also notice and allow students to struggle with the counter intuitive principle of multiplying a whole number by a fractional number where the product gets smaller rather than larger.

Have concrete material available if needed for students to select (e.g., fraction tiles).

Expect students to represent using drawings, number-line or fraction pieces to represent parts of the whole and explain these using notation.

Shareback

Select students who either use repeated addition for the fractional numbers (e.g., add $\frac{5}{6}$ 11 times) or solve the problem as 11 x $\frac{5}{6}$ = $\frac{55}{6}$ or $9\frac{1}{6}$. If the second multiplicative solution is not used, then model as another way a student has used previously.

Big Ideas

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On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit (2 x $\frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3}$ x 2); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Connect

$\frac{55}{6} = ?$

$\frac{14}{3} = ?$

$\frac{45}{4} = ?$

What patterns do you notice when changing an improper fraction to a mixed number?

Suggested Learning Outcomes

Multiply a fraction by a whole number.

Convert improper fractions to mixed numbers.

Independent Tasks

Dion is preparing bags of apples to sell at the local market. Each bag has $1\frac{1}{2}$ of a kilogram of mandarins. Dion needs to prepare 20 bags to sell, how many kilograms of apples does he need to fill the bags?

A bike travels $\frac{2}{5}$ of a kilometre in one minute. How many kilometres will the bike have travelled after 23 minutes?

A recipe to make 20 pieces of fudge needs $\frac{7}{8}$ of a cup of sugar. Stella wants to make enough fudge for her hub at school which has 120 students. How much sugar does Stella need?

Curriculum Links

During Year Seven and Eight

add and subtract fractions with different denominators, using equivalent fractions

multiply fractions and decimals by whole numbers

Mathematical Language

Whole, half, halves, quarters, fourths, thirds, equivalent, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 7

Ilapesi and Sela are preparing ‘otai for the students in their class for Tongan language week. To make enough ‘otai for four people, they need:

$1\frac{3}{4}$ cups of pineapple
$3\frac{1}{2}$ cups of watermelon
$\frac{5}{6}$ cup of coconut
$\frac{4}{5}$ cup of coconut milk
1 Lemon
$\frac{7}{8}$ cup of coconut water
$\frac{2}{3}$ cup of lime
$2\frac{2}{5}$ cups of ice cubes

They want to make enough ‘otai for 32 people. Write a list of the total amount of each ingredient that they will need.

Teacher Notes

Facilitate the students to notice that there are multiples of the fractional number which they can add or multiply

Notice students who use multiplicative thinking. Also notice and allow students to struggle with the counter intuitive principle of multiplying a whole number by a fractional number where the product gets smaller rather than larger.

Have concrete material available if needed for students to select (e.g., fraction tiles).

Expect students to represent using drawings, number-line or fraction pieces to represent parts of the whole and explain these using notation.

Shareback

Select students who either use repeated addition for the fractional numbers (or solve the problem as multiplication. If the second multiplicative solution is not used, then model as another way a student has used previously.

Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

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Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also $26 \times 4 = (20 + 6) \times 4$).

Connect

$\frac{2}{5}$ x 8 =

$\frac{4}{9}$ x 12 =

$\frac{7}{10}$ x 9 =

What patterns do you notice when multiplying fractions?

Suggested Learning Outcomes

Multiply a fraction by a whole number.

Convert improper fractions to mixed numbers.

Independent Tasks

Gabriella filled 5 glasses with $\frac{2}{3}$ of a litre of sprite into each glass.
How much sprite did Gabriella use?

You have $\frac{3}{4}$ of a pizza left. If you give $\frac{1}{3}$ of the pizza to your cousin, how much of a whole pizza will your cousin get?

Louise used $2\frac{1}{2}$ tubes of paint. Each tube holds $\frac{4}{5}$ gram of paint.
How many grames of paint did Louise use?

Curriculum Links

During Year Seven and Eight

add and subtract fractions with different denominators, using equivalent fractions

multiply fractions and decimals by whole numbers

Mathematical Language

Whole, thirds, tenths, equal, equivalent, numerator, denominator, common denominator, common multiple.

Anticipations

Solutions, Misconceptions

Task 8

You are helping to make ula lole for a celebration. There are bags of fruit bursts to use to make each ula lole.

Your Mum gives you 4 and $\frac{3}{4}$ bags of fruit bursts.

You find that on average you use $\frac{2}{3}$ of a bag of fruit bursts to make each ula lole.

You use up all the fruit bursts you have. How many ule lole have you made altogether?

Teacher Notes

Facilitate the students to notice that there are multiples of the fractional number which they can subtract repeatedly.

Notice students who use relationships to solve these word problems. For example, most students will repeatedly add or subtract but notice the students who see the relationship as groups of in multiplicative ways.

Monitor for students who make the conjecture that when dividing by a fractional number the dividend gets bigger rather than smaller as it does when dividing by a whole number. Record and explore with students.

In the connect, press students to consider division of fractions as how many times something can go into...be subtracted.

Shareback

Select students to who use measurement division (repeated subtraction as division,

e.g., $4\frac{3}{4} - \frac{2}{3} - \frac{2}{3} - \frac{2}{3}$ ...)

or who use the inverse relationship of multiplication and division

($\frac{2}{3}$ x ? = 4) or ($\frac{2}{3}+\frac{2}{3}+\frac{2}{3}$... = 4). If either solution is not used, then model as another way the teacher has seen used previously.

Big Ideas

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Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3}$ x $\frac{1}{4}$; also 26 x 4 = (20 + 6) x 4).

Connect

Ask students to describe how you would solve the following problems using division (repeated subtraction) or multiplication (repeated addition):
It takes $\frac{1}{6}$ of a bag of minties for 1 ula lole.

I have 1 bag of minties, how many ula lole can I make?
I have 2 bags of minties, how many ula lole can I make?
I have 10 bags of minties, how many ula lole can I make?
I have a $\frac{1}{2}$ bag of minties, how many ula lole can I make?

I have a 2 $\frac{1}{2}$ a bag of minties, how many ula lole can I make?

What pattern do you notice?
Could you describe what you are doing when you are dividing by a fraction?

Suggested Learning Outcomes

Divide a mixed number by a fraction.

Use repeated subtraction to solve division problems.

Independent Tasks

You are helping to make ula lole for a celebration. There are bags of fruit bursts to use to make each ula lole.

Your Mum gives you 4 and $\frac{5}{6}$ bags of fruit bursts.
You find that on average you use $\frac{4}{5}$ of a bag of fruit bursts to make each ula lole.
You use up all the fruit bursts you have. How many ule lole have you made altogether?
Record an explanation to tell someone else what is important about dividing fractions.

You are helping to make ula lole for a celebration. There are bags of fruit bursts to use to make each ula lole.
Your Mum gives you 8 and $\frac{3}{4}$ bags of fruit bursts.
You find that on average you use $\frac{7}{9}$ of a bag of fruit bursts to make each ula lole.
You use up almost all the fruit bursts you have. How many ule lola have you made altogether?
Record an explanation to tell someone else what is important about dividing fractions.

Curriculum Links

During Year Seven and Eight

add and subtract fractions with different denominators, using equivalent fractions

multiply fractions and decimals by whole numbers

Mathematical Language

Whole, half, halves, quarters, fourths, whole number, equivalent, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 9

Koro has $1\frac{1}{5}$ hours to do 3 things to help his family around his home.

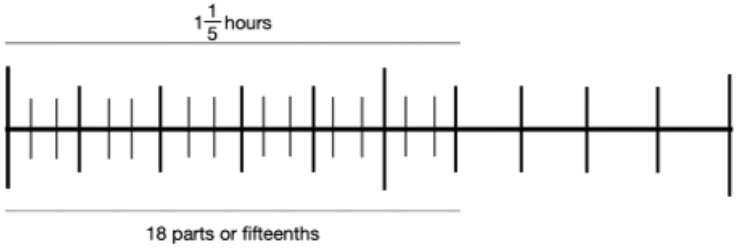
How much time does he take to do each job?
Solve this problem using both a number line and counters.
Now test your answer against the solution in minutes.

Teacher Notes

During the launch, reinforce that the students can work out the solution using minutes, but they also need to use both a number line and counters as representations of their reasoning.

Have counters and empty number lines available.

Facilitate the students to notice that one approach is to divide each fifth into 3 parts. Number line representation is as follows:



To represent with counters, they need to divide each fifth into three parts. They use a set of 15 counters as a whole in an array of 5 lots of 3 and a second array of 3 for the part of the next hour so that they have

represented $1\frac{1}{5}$ hours. They have 18 counters in all.

Shareback

Select students to share who are able to justify their explanation using number lines and counters to represent their solution.

Big Ideas

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also $26 \times 4 = (20 + 6) \times 4$).

Connect

What fraction would you need to work with if he had:

2 jobs in $2\frac{1}{7}$ hours?

4 jobs in $3\frac{1}{5}$ hours?

7 jobs in $1\frac{1}{2}$ hours?

What patterns do you notice?

Suggested Learning Outcomes

Divide a mixed number by a fraction.

Use repeated subtraction to solve division problems.

Independent Tasks

Solve these equations:

$\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}+\frac{2}{3}=\_\_ \times \_\_ = \_\_$

$4 \times \_\_ = \frac{5}{6}+\frac{5}{6}+\frac{5}{6}+\frac{5}{6}=$

$\frac{4}{7}+?=2\times\frac{4}{7}=$

$\frac{3}{8}\times\_\_ = \frac{\square}{\square}+\frac{\square}{\square}+\frac{\square}{\square}$

$3 \div \frac{1}{2} =$

Write a story problem that would match these equations and solve them:

$\frac{4}{5}\times 6 =$

$\frac{2}{3}+\frac{7}{9}=$

$6 \div \frac{2}{7} =$

$5 \div \frac{5}{6} =$

Curriculum Links

During Year Seven and Eight

add and subtract fractions with different denominators, using equivalent fractions

multiply fractions and decimals by whole numbers

Mathematical Language

Whole, fourths, quarters, equal, equivalent, numerator, denominator, common denominator, common multiple.

Anticipations

Solutions, Misconceptions

Task 10

Mum is filling up the petrol tank.

The petrol tank is $\frac{1}{3}$ full and she pumps 35 litres into the tank.

Now the tank is $\frac{4}{5}$ full.

How many litres would be needed to fill the tank completely?

Make sure you are able to justify your explanation using both representations and notation.

Teacher Notes

During the launch, facilitate students to notice that whole number problems and fractional number problems may be start unknown or change unknown problems and not result unknown only.

Expect the students to represent their reasoning using both diagrams (rectangular box for fractions), number-lines, and notation.

Shareback

Select students to share who represented the problem as $\frac{1}{3}$ + [35] = $\frac{4}{5}$ and who have used a common multiple and denominator for third and fifths to solve the problem along with representation.

Connect

Make statements of things that you know are always true about fractions. Explore and prove these.

Suggested Learning Outcomes

Find and compare equivalent fractions.

Add and subtract fractions.

Find fractions of a set.

Big Ideas

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b}$ = a ÷ b, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3}$ = 2 ÷ 3.

On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit (2 x $\frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3}$ x 2); each is associated with the same point on the number line..

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also 26 x 4 = (20 + 6) x 4).

Independent Tasks

Marika has been given a cake to decorate. She is given 108 lollies to decorate it. She decides to split the cake into three sections and decorate each section but with a different proportion of lollies on each section.
She puts $\frac{4}{6}$ of her lollies on the first section.
She puts $\frac{2}{9}$ of the lollies on the second section.

She puts $\frac{1}{12}$ of the lollies on the third section.

How many lollies does she put on each section?

Marika is making otai. For each bottle of otai she needs:

1 and $\frac{1}{2}$ cups of pineapple.

3 and $\frac{2}{3}$ cups of watermelon.

1 and $\frac{3}{4}$ of a cup of coconut milk.

$\frac{5}{6}$ of a cup of coconut water.

$\frac{1}{4}$ of a cup of lemon juice.

Marika wants to make 6 bottles of otai. How much of each ingredient will she need?

Curriculum Links

During Year Seven and Eight

find a percentage of a whole number, and find a whole amount, given a simple fraction or percentage (e.g., "75% is $45, what is the total amount?")

Mathematical Language

Whole, thirds, ninths, tenths, fifths, equal, equivalent, fair share, partitioning, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 11

If 72 felts is $2\frac{1}{4}$ sets. How much is 1 set of felts?

If you run fast, you can cover $3\frac{1}{2}$km in $\frac{3}{4}$ of an hour.
How fast are you running in kilometres per hour?

Teacher Notes

Facilitate the students to notice the importance of considering the whole.

Notice students who use reasoned notation to solve the equations.

Shareback

Select students to share who have reasoned through using the amount of one fourth (equal partitioning) and then the value of one whole (iterating)

Connect

In solving these two problems you had to keep thinking: How much is one?
Can you explain why this was important?

Suggested Learning Outcomes

Divide a fraction by a fraction.

Independent Tasks

Can you write 3 fraction multiplication problems and then solve them?

Can you write 3 fraction division problem and then solve them?

Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

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Anticipations

Solutions, Misconceptions

Curriculum Links

During Year Seven and Eight

find a percentage of a whole number, and find a whole amount, given a simple fraction or percentage (e.g., "75% is $45, what is the total amount?")

Mathematical Language

Whole, fifths, thirds, sevenths, tenths, sixths, equal, equivalent, numerator, denominator, common denominator, common multiple.

Task 12

Solve each equation.

1. $a \times \frac{1}{4} = 2$
2. $3 \times \frac{1}{3} + 2 \times \frac{1}{3} = b$
3. $8 \times \frac{3}{8} = c$
4. $d \times \frac{3}{4} = 15$
5. $e \times 1\frac{2}{3} = 25$
6. $\frac{1}{3} = f \times \frac{1}{6}$
7. $\frac{2}{5} = g \times \frac{1}{10}$
8. $\frac{3}{8} + h = \frac{1}{2}$
9. $\frac{7}{20} = \frac{i}{15}$
10. $\frac{1}{10} = 2 \times j$
11. $\frac{8}{5} = k \times \frac{1}{10}$
12. $2 \times \frac{1}{3} = l \times \frac{1}{6}$

Be ready to explain and justify your explanations using representations and/or notation.

Teacher Notes

Facilitate the students to draw on the relationships they have identified in addition, subtraction, multiplication and division.

Notice students who are able to generalise patterns across denominators.

Shareback

Select students to share who are able to draw on number properties and operational laws including the understanding of the equals sign to solve the problems.

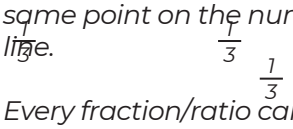
Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b ≠ 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.



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The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Connect

Draw on student solution strategies to highlight operational laws that the students used to solve the problems (e.g., inverse relationship, commutative property).

Connect back to number and algebra unit to highlight that the properties that work with all numbers including fractions.

Suggested Learning Outcomes

Solve missing number problems that involve fractions.

Solve addition problems involving fractions.

Solve subtraction problems involving fractions.

Solve multiplication problems involving fractions.

Solve division problems involving fractions.

Use operational laws to solve missing number problems that involve fractions.

Independent Tasks

Choose from one of the following Assessment tasks:

Task 1: Fractions/Proportion, Ratio and Decimals - Write an equation.

Task 2: Order Fractions

Task 3: Fractions (set)

Curriculum Links

During Year Seven and Eight

multiply fractions and decimals by whole numbers

During Year Seven
describe and use the commutative, distributive, and associative properties of operations (e.g., $a \times b = b \times a$)

During Year Eight
simplify algebraic expressions involving sums, products, differences, and single brackets (e.g., using the distributive property, $2(x + 3) + 1 = 2x + 6 + 1 = 2x + 7$)

Mathematical Language

Whole, half, halves, eighths, whole number, equal, equivalent, section, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Assessment Task 1 - Fractions - Year 7 - 8

Write some word problems for a friend involving any of the operations (addition, subtraction, multiplication, division) using fractions, decimals, or percentages. Show how you would solve the problems.

Assessment Task 1 - Fractions - Year 7 - 8

Put these fractions in order from smallest to biggest.

$\frac{9}{15}$ $\frac{7}{12}$ $\frac{8}{10}$ $\frac{2}{3}$ $\frac{14}{16}$ $\frac{5}{6}$

Explain and show how you know this.

Assessment Task 2 - Fractions - Year 7 - 8

Solve these problems and prove and justify your solution:

1) Mum fills up the petrol tank. During the week, she uses $\frac{1}{4}$ of the petrol dropping the kids off at school. She uses $\frac{3}{8}$ of the petrol going to work. She uses $\frac{1}{6}$ of the petrol going to the supermarket. What fraction of the petrol tank is full now?

2) Jodie is selling mandarins from her tree. She is selling them in $\frac{2}{3}$ kg bags. If she picks a bucket of mandarins weighing 5 and $\frac{5}{6}$ kgs – how much of the bags will she be able to fill?

3) Talia is making a fruit smoothie. For each bottle she needs:

$\frac{3}{4}$ of a cup of blueberries $\frac{2}{3}$ of a cup of banana $\frac{4}{5}$ of a cup of ice

$2\frac{3}{4}$ cups of milk $\frac{1}{5}$ of a table-spoon of vanilla

Talia wants to make 5 bottles. How much of each ingredient will she need?