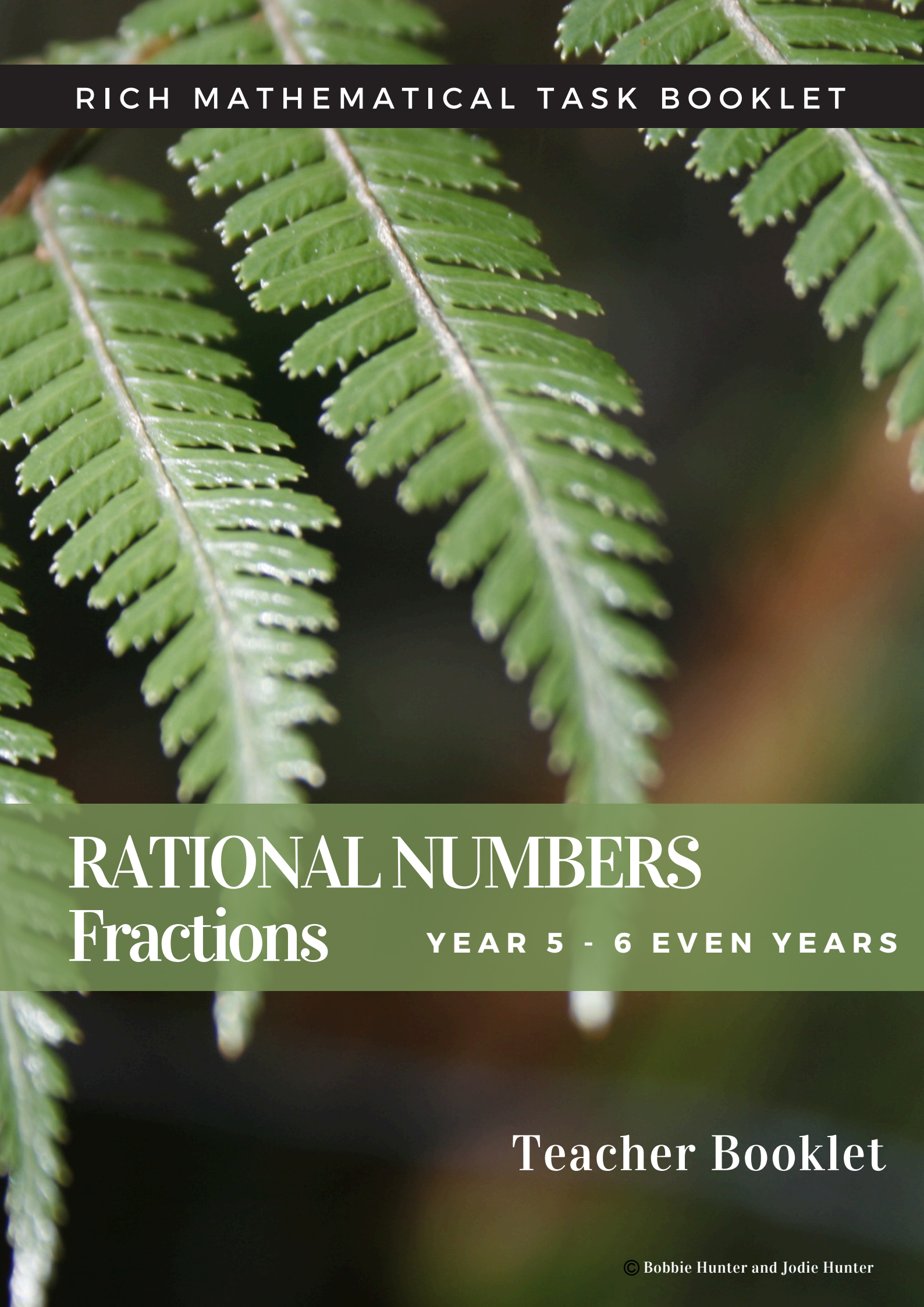

RICH MATHEMATICAL TASK BOOKLET

RATIONAL NUMBERS
Fractions

YEAR 5 - 6 EVEN YEARS

Teacher Booklet

Task 1

Use the fraction tiles to make one whole in different ways.
Record your responses.
[Complete first part of the connect below after this part of the lesson].

Use the fraction tiles to make different fractional numbers that are more than one whole but less than one and a third. Record your responses.

Use the fraction tiles to make different fractional numbers that are less than one half but more than two twelfths.. Record your responses.

Use the fraction tiles to make different fractional numbers that are more than one whole but less than one and a quarter. Record your responses.

Use the fraction tiles to make different fractional numbers that are less than one half but more than one quarter. Record your responses.

Teacher Notes

Before the launch have the students explore and talk together about the fraction pieces for a whole, halves, quarters, eighths, thirds, sixths, twelves. Work through each task and discuss before moving to the next task.

Have fraction tiles available for the task.

Monitor for students using the words fractional numbers (not pieces or bits) and greater than, less than, the same as.

Notice students who make generalisations (e.g., the smaller the denominator the bigger the fraction when the numerator is one). Record these as class conjectures and have students explore and prove at a later date as a warm-up activity.

Expect students to represent materials and use appropriate notation and the equal sign ($\frac{2}{2} = 1$).

For the independent task, you will need fraction tile sets available for students who would like to use them.

Shareback

Select students to share who made combinations of the whole or other fractions using the same size pieces (halves, quarters, sixths, eighths) and recorded the combinations as numbers, equations, or words. Then, select students to share who made combinations of the whole using the unlike fraction pieces (e.g., $\frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$) and recorded the combinations as numbers, equations or words.

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Connect

Complete this part of the connect after the first part of the task.

What is $\frac{4}{4}$ the same as?
What is $\frac{50}{50}$ the same as?
What is $\frac{1000}{1000}$ the same as?
What patterns and relationships do you notice?

What other fractions are the same as one whole?
[Encourage students to record using equals sign $\frac{2}{2} = \frac{5}{5} = \frac{100}{100}$]
What is a rule for fractions that equal one whole?
[Record conjectures and symbolise as $\frac{n}{n}$]

What other fractions are the same as one half?
[Encourage students to record using equals sign $\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$]
What patterns and relationships do you notice?
What is the rule to know whether fractions equal one half?

Suggested Learning Outcomes

Combine and recombine different units of fractions to make one whole.

Identify and recognise equivalent fractions.

Recognise and use improper fractions to represent more than one whole.

Independent Tasks

What other fractions are the same as one whole?
Record these using at least three different representations (number line, drawings, equations).

What other fractions are the same as one third?
Record these using at least three different representations (number line, drawings, equations).

What other fractions are the same as one quarter?
Record these using at least three different representations (number line, drawings, equations).

What other fractions are the same as one eighth?
Record these using at least three different representations (number line, drawings, equations).

Curriculum Links

During Year Five
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Add and subtract fractions with the same denominators, including to make more than one whole

During Year Six
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Add and subtract fractions with the same or related denominators

Mathematical Language

Whole, half, halves, quarters, fourths, thirds, sixths, tenths, twelfths, eighths, fraction, fractional number, whole number, equal, equivalent, greater than, less than.

Anticipations

Solutions, Misconceptions

Task 2

Tupou says that she can write more than 20 numbers between 0 and 1. Hemi says that there are none, so Tupou writes them and uses a number-line to prove that they exist.

Record some numbers that you think Tupou wrote and show where you think she marked them on her number-line. Make the numbers with the fraction tiles.

Teacher Notes

Before you launch the task, provide students with fraction tiles and ask them to complete the following tasks:

Use the fraction tiles to make different fractional numbers that are the same as one fifth.

Use the fraction tiles to make different fractional numbers that are between two thirds and seven eighths..

Have large sheets of paper and pens for the students to draw their number-lines.

When you launch the task, facilitate students to note that they should be writing fractions for this task rather than decimals.

Facilitate the students to notice that earlier in the year, the number lines they have used only contained whole numbers (numbers that resulted from counting). The fractions they are talking about now (numbers resulting from equal splitting or partitioning) can be represented on the number line. This shows that fractions may also be thought of as numbers. In the connection refer to the fractions (e.g., $3\frac{1}{2}$ as a mixed number.

Monitor for students using vocabulary which emphasises dividing or splitting equally of portioning into equal parts.

Notice students who find the concept of fractions as numbers between numbers counter intuitive. Allow them to struggle and construct reasoning through mathematical talk and using agreeing mathematically and disagreeing mathematically (e.g., I agree because...).

For the independent task, provide the students with the fraction tiles.

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Each fraction can be associated with a unique point on a number line.

There is no least or greatest fraction on the number line.

There are an infinite number of fractions between any two fractions on the number line

Shareback

Select students to share who can visualise and draw a number-line and mark the positions of fractions between 1 and 2 by partitioning the number-line.

Connect

What mixed numbers are there between 1 and 4?
Mark each one on a number-line to show where they would be.

Suggested Learning Outcomes

Record fractions on a number-line.

Recognise there are an infinite number of fractions between any two whole numbers or any two fractions on a number line.

Combine and recombine different units of fractions to make one whole.

Identify and recognise equivalent fractions.

Independent Tasks

Draw a number-line starting from 0 and finishing at 10. Put at least 15 different fractions on the number-line.

Draw a number-line starting from 0 and finishing at 5. Put at least 15 fractions on the number-line. Don't use the same fractions as you used previously.

Draw a number-line starting from 0 and finishing at 2. Put at least 10 fractions on the number-line. Don't use the same fractions as you used previously.

Use the fraction tiles to explore the following questions:

Is $\frac{4}{6}$ of a piece of chocolate bar the same as $\frac{2}{3}$ of a piece of chocolate bar? Why or why not?

Is $\frac{3}{4}$ of a piece of chocolate bar the same as $\frac{4}{8}$ of a piece of chocolate bar? Why or why not?

Is $\frac{1}{2}$ of a piece of chocolate bar the same as $\frac{2}{4}$ or $\frac{3}{6}$ or $\frac{4}{8}$ of a piece of chocolate bar?
Why or why not?

Is $\frac{9}{10}$ of a piece of chocolate bar bigger than $\frac{4}{5}$ of a piece of chocolate bar? Why or why not?

Curriculum Links

During Year Five
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Convert between mixed numbers and improper fractions with denominators of up to 10

During Year Six
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Convert between mixed numbers and improper fractions

Mathematical Language

Whole, half, halves, quarters, fourths, thirds, sixths, twelfths, eighths, fraction, whole number, equal, equivalent, mixed numbers, equivalent, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 3

Who eats more?

1. Five people sharing 4 chocolate bars equally.
2. Three people sharing 2 chocolate bars equally.
3. Four people sharing 3 chocolate bars equally.
4. Six people sharing 5 chocolate bars equally
5. Eight people sharing 7 chocolate bars equally.

Put them in order from smallest to largest. Be ready to explain and justify your reasoning in multiple ways.

Teacher Notes

During the launch, model for the students that the chocolate bar should be represented as a rectangular shape.

Notice students who use the language of comparison and because as part of their justification.

Expect students to represent using real life contexts (e.g., if you were comparing slices of a cake and who had more or less then $\frac{7}{8}$ is closer to the whole cake because $\frac{1}{8}$ slice is a smaller slice than $\frac{1}{5}$).

In the connect, students may generalise that the larger the denominator the smaller the fraction, however, facilitate them to notice that this only applies to unit fractions and is not a rule for all fractions.

Shareback

Select students to share who develop representations to justify their reasoning and either split all the chocolate bars into the fractional amounts or share as a whole and fractional amount.

If the second solution is not used, then model as another way the teacher has seen used previously.

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Connect

Record the matching equations for each problem and model the first two then ask students to solve the rest.

$7 \div 6 = \frac{7}{6} = 1\frac{1}{6}$

$8 \div 5 = \frac{8}{5} = 1\frac{3}{5}$

$3 \div 4 =$

$6 \div 4 =$

$9 \div 8 =$

$15 \div 10 =$

What patterns and relationships do you notice that can help you solve the problems?

Suggested Learning Outcomes

Divide a whole number into fractions.

Compare and use different sized fractions.

Recognise and use improper fractions to represent more than one whole.

Independent Tasks

At the speed chocolate eating contest each contestant has to eat as much of a chocolate bar as they can in 15 seconds. These are the results of how much of 1 chocolate bar each contestant managed to eat:

Daniel – $\frac{2}{3}$ Leti – $\frac{7}{9}$ Georgie – $\frac{2}{5}$

Sose – $\frac{10}{16}$ Talasi – $\frac{3}{4}$ Jeni – $\frac{1}{2}$

Can you put the results in order – from who ate the most chocolate to who ate the least? Try and prove your answer in a number of different ways.

Curriculum Links

During Year Five

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Convert between mixed numbers and improper fractions with denominators of up to 10

During Year Six

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Convert between mixed numbers and improper fractions

Mathematical Language

Whole, quarters, fourths, thirds, sixths, twelfths, eighths, ninths, fifteenths, fraction, equal, equivalent, mixed numbers, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 4

Nixon is playing NBA 2K league on the PS4. He wants to buy a player with the best field goal percentage. He has the following options:

For every eight shots, LeBron James scores 4.
For every five shots, James Harden scores 2.
For every ten shots, Anthony Davis scores 5.
For every three shots, Hassan Whiteside scores 2.
For every twelve shots, Matthew Mooney scores 3.

Rank the players in order from the best buy to the worst buy.

Prove your answer in a variety of ways using drawings, words and numbers.

Teacher Notes

For the conceptual starter activity, ask the students to put the following numbers on a number line on the board showing:

0 1 2 3 4

$\frac{1}{2}$ $\frac{3}{6}$ $2\frac{1}{2}$ $2\frac{3}{4}$ $3\frac{2}{5}$ $3\frac{7}{8}$

Provide students with fraction tiles as needed. These can also be used to model how to find equivalent fraction and show the relationships.

Monitor for students using vocabulary of equivalence and relational thinking. Students may notice that to compare fractions accurately the denominators need to be the same.

Expect students to represent using notation and drawings to justify equivalences.

Shareback

Select students to share who converted fractions to equivalent fractions using informal methods with representations before they compared the fractions.

If any students changed all the fractions to a common denominator than select them to share last.

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Connect

Which is bigger?

$\frac{3}{5}$ or $\frac{7}{10}$

$\frac{2}{3}$ or $\frac{5}{6}$ or $\frac{8}{12}$

What patterns and relationships did you use to find equivalent fractions to compare them?
Can you come up with a rule to change fractions but keep them equivalent?

Introduce the concept that you can multiply the denominator by a number and as long as you also multiply the numerator by the same number it will be equivalent.

Suggested Learning Outcomes

Represent fractions.

Compare and order fractions.

Find equivalent fractions.

Independent Tasks

The soccer team all have the same sized cups. Throughout the game this is how much they drank:

Tayla drinks five quarters of a cup.

Loni drinks three halves of a cup.

Tere drinks five thirds of a cup.

Mia drinks ten eighths of a cup.

Put how much they drank in order from most to least.

Prove your solution using at least 3 different representations.

Curriculum Links

During Year Five
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Convert between mixed numbers and improper fractions with denominators of up to 10

During Year Six
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Convert between mixed numbers and improper fractions

Mathematical Language

Whole, quarters, fourths, thirds, sixths, twelfths, eighths, ninths, fifteenths, fraction, equal, equivalent, mixed numbers, greater than, less than, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 5

Timo and Sesimani are having an argument about the solution to this maths problem:

Loti had two chocolate bars. She ate $\frac{1}{3}$ of one bar and $\frac{5}{6}$ of the second bar. She gave the rest to her sister. What fraction of the chocolate bars did Loti eat?

Timo says that Loti ate $1\frac{1}{6}$ of the chocolate bars.

Sesimani says that Loti ate $\frac{6}{9}$ of the chocolate bars.

Who is correct and why?
Develop an explanation for the correct answer and how you could prove this in different ways.

Teacher Notes

Facilitate the students to notice that to add fractions the denominators need to be the same.

Monitor for students using vocabulary of equivalence and relational thinking.

Notice students who show relational understanding ($\frac{3}{4}$ as $\frac{1}{2}+\frac{1}{4}$) or who use equivalence relationships).

Expect students to represent using drawings and notation.

Shareback

Select students to share who converted fractions to equivalent fractions using informal or more formalised methods (multiplication for example) before they added the fractions.

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Connect

What would be a common denominator if you were adding:

$\frac{1}{4}$ and $\frac{1}{8}$
$\frac{1}{2}$ and $\frac{1}{7}$
$\frac{1}{3}$ and $\frac{1}{5}$
$\frac{1}{6}$ and $\frac{1}{18}$
$\frac{2}{3}$ and $\frac{1}{4}$
$\frac{5}{6}$ and $\frac{1}{5}$

Can you find a pattern for finding a common denominator?

Suggested Learning Outcomes

Change fractions to equivalent fractions.

Solve problems that involve adding fractions.

Independent Tasks

Litea and her two friends are at the movies.
They each buy a big tub of popcorn.
Litea eats $\frac{3}{4}$ of her tub.
Kaia eats $\frac{2}{3}$ of his tub.
Gaylene eats $\frac{1}{2}$ of her tub.
They tip all the left-over popcorn into two tubs.
How much is left to take home?

Curriculum Links

During Year Five
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Add and subtract fractions with the same denominators, including to make more than one whole

During Year Six
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Add and subtract fractions with the same or related denominators

Mathematical Language

Whole, quarters, fourths, thirds, sixths, equal, equivalent, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 6

Jean and her friends are making some clay beads for necklaces. They have 3 packs of modelling clay.

Henry uses $\frac{1}{4}$ of a pack of modelling clay.
Tupou uses $\frac{2}{5}$ of a pack of modelling clay.
Claire uses $\frac{5}{6}$ of a pack of modelling clay.
Jean uses the rest.

How much does Jean have to make her clay beads?

Teacher Notes

For the conceptual starter activity, ask the students to put the following numbers on a number line on the board showing:

0 1 2 3 4

$3\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{8}$ $2\frac{2}{3}$ $1\frac{1}{6}$ $1\frac{4}{8}$

Facilitate the students to notice that to add fractions the denominators need to be the same.

Monitor for students using vocabulary of equivalence and relational thinking.

Notice students who show relational understanding ($\frac{3}{4}$ as $\frac{1}{2} + \frac{1}{4}$) or who use equivalence relationships).

Expect students to represent using drawings and notation.

Shareback

Select students to share who converted fractions to equivalent fractions using informal or more formalised methods (multiplication for example) before they added the fractions.

Big Ideas

Numbers can be described in many different ways including as fractions.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{1}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Connect

What are the common denominators between these numbers?

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

$\frac{1}{2}, \frac{1}{5}, \frac{1}{10}$

Can you find a pattern?
What rule could you use to find common denominators?

Suggested Learning Outcomes

Find equivalent fractions.

Subtract a fraction from a whole number.

Add and subtract fractions.

Generalise how to find common denominators.

Independent Tasks

Find the solutions.

Selena has $\frac{1}{2}$ of a bag of marbles. Luke has $\frac{1}{4}$ of a bag of marbles.
How much of a bag of marbles do they have altogether?

Selena has $\frac{1}{3}$ of a bag of marbles. Luke has $\frac{1}{6}$ of a bag of marbles.
How much of a bag of marbles do they have altogether?

Selena has $\frac{1}{4}$ of a bag of marbles. Luke has $\frac{1}{3}$ of a bag of marbles.
How much of a bag of marbles do they have altogether?

Selena has $\frac{1}{2}$ of a bag of marbles. Luke has $\frac{1}{5}$ of a bag of marbles.
How much of a bag of marbles do they have altogether?

Selena has $\frac{3}{4}$ of a bag of marbles. Luke has $\frac{1}{5}$ of a bag of marbles.
How much of a bag of marbles do they have altogether?

Selena has $\frac{7}{8}$ of a bag of marbles. Luke has $\frac{1}{3}$ of a bag of marbles.
How much of a bag of marbles do they have altogether?

Curriculum Links

During Year Five
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Add and subtract fractions with the same denominators, including to make more than one whole

During Year Six
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Add and subtract fractions with the same or related denominators

Mathematical Language

Whole, half, halves, quarters, fourths, thirds, equivalent, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 7

Alisi's aunties are making a fine Tongan mat.
Aunty Seini uses $\frac{1}{2}$ of a ball of red wool.
Aunty Hiva uses $\frac{1}{3}$ of the red wool.
How much more wool does Aunty Seini use?

Alisi's aunties are making a fine Tongan mat.
Aunty Seini uses $\frac{7}{8}$ of a ball of red wool.
Aunty Hiva uses $\frac{1}{3}$ of the red wool.
How much more wool does Aunty Seini use?

Alisi's aunties are making a fine Tongan mat.
Aunty Seini uses $\frac{9}{10}$ of a ball of red wool.
Aunty Hiva uses $\frac{5}{6}$ of the red wool.
How much more wool does Aunty Seini use?

Teacher Notes

Have fraction strips and number-lines available.

Facilitate the students to notice that to subtract fractions they need to find a common denominator.

Monitor for students using vocabulary of equivalence and relational thinking.

Shareback

Select students to share who converted fractions to equivalent fractions using informal or more formalised methods (multiplication for example) before they subtracted the fractions.

Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also $26 \times 4 = (20 + 6) \times 4$).

Connect

What is the difference between:

$\frac{1}{2}$ and $\frac{3}{4}$

$\frac{2}{5}$ and $\frac{3}{10}$

$\frac{7}{8}$ and $\frac{3}{4}$

$\frac{1}{5}$ and $\frac{1}{3}$

$\frac{1}{7}$ and $\frac{1}{8}$

What patterns did you use to solve these in your mind?

Suggested Learning Outcomes

Find equivalent fractions.

Subtract a fraction from a fraction.

Generalise how to find common denominators.

Independent Tasks

Two fractions add to give $\frac{1}{2}$. What might those fractions be? Give a range of answers.

A friend of mine put these fractions into two groups but they got mixed up. What might the two groups be?

$\frac{1}{5}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{8}{12}$ $\frac{5}{16}$ $\frac{2}{8}$

What might the missing fraction be?

__ < $\frac{3}{4}$

__ + __ = $\frac{2}{5}$

$\frac{1}{?} = \frac{?}{?}$

Curriculum Links

During Year Five

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Add and subtract fractions with the same denominators, including to make more than one whole

During Year Six

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Add and subtract fractions with the same or related denominators

Mathematical Language

Whole, half, halves, thirds, sixths, eighths, tenths, equal, equivalent, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 8

Malia is making otai. For each jug of otai she needs:

Two and a quarter cups of pineapple.
Three and half cups of watermelon.
Three quarters of a cup of coconut milk.
Three quarters of a cup of coconut water.
One quarter of a cup of lemon juice.

Malia wants to make 9 jugs of otai. How much of each ingredient will she need?

Teacher Notes

During the launch, establish the context of the problem. Otai is a dish common to the Pacific. The recipe for this otai is Tongan.

Facilitate the students to notice that there are multiples of the fractional number which they can add or multiply

Notice students who use multiplicative thinking. Also notice and allow students to struggle with the counter intuitive principle of multiplying a whole number by a fractional number where the product gets smaller rather than larger.

Have concrete material available if needed for students to select (e.g., fraction tiles). Expect students to represent using drawings, number-line or fraction pieces to represent parts of the whole and explain these using notation.

Shareback

Select students who either use repeated addition for the fractional numbers (e.g., add $\frac{1}{4}$ nine times for the lemon juice and get $\frac{9}{4}$); or add the fourths and get nine fourths and see this as equivalent to two wholes and one fourths; or solve the problem as 9 x = $\frac{1}{4}$ or $2\frac{1}{4}$ If the second multiplicative solution is not used, then model as another way the teacher has seen used previously.

Big Ideas

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The real-world actions for addition and subtraction of whole numbers are the same for operations with fractions and decimals.

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also $26 \times 4 = (20 + 6) \times 4$).

Connect

$\frac{1}{2}$ x 4 =

$\frac{1}{2}$ x 8 =

$\frac{1}{4}$ x 8 =

$\frac{1}{4}$ x 20 =

$\frac{1}{3}$ x 6 =

$\frac{1}{10}$ x 50 =

What patterns do you notice when you are multiplying fractions?

Suggested Learning Outcomes

Multiply a mixed number by a whole number.

Multiply a fraction by a whole number.

Independent Tasks

Malia is making otai. For each bottle of otai she needs:

1 and $\frac{3}{4}$ cups of pineapple.
2 and $\frac{1}{2}$ cups of watermelon.
1 and $\frac{1}{4}$ of a cup of coconut milk.
Three quarters of a cup of coconut water.
$\frac{1}{2}$ of a cup of lemon juice.

Malia wants to make 6 bottles of otai. How much of each ingredient will she need?

Curriculum Links

During Year Five
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent

Add and subtract fractions with the same denominators, including to make more than one whole

Use known multiplication facts to scale a quantity

During Year Six
For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:
– compare and order the fractions
– identify when two fractions are equivalent
– represent the fractions in their simplest form

Add and subtract fractions with the same or related denominators

Use known multiplication and division facts to scale a quantity

Mathematical Language

Whole, half, halves, quarters, fourths, whole number, equivalent, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 9

Mereana is making a picture frame using New Zealand shells. She uses 40 pieces of paua shell, 200 pieces of spiral shells and 88 cockle shells.

For her first draft she splits her frame into 4 sections. How many of each shell does she use on each section?

For her second draft she splits her frame into 3 sections. How many of each shell does she use on each section? How many does she have left over?

For her third draft she splits her frame into 7 sections. How many of each shell does she use on each section? How many does she have left over?

Teacher Notes

During the launch, ensure that you reinforce that each set of shells are one whole as part of developing the context.

Facilitate the students to notice that they are finding a fraction of a whole even when there are a number of items in that set. Also, draw attention to the denominator as naming what the whole is divided into.

Monitor for students using vocabulary of the whole and parts of the set.

Expect students to use chunking or division to solve the fraction tasks.

Shareback

Select students to share who have used a representation split into fraction parts and then have either used partitive division (e.g., 200 ÷ 4 = ?) or have used the inverse relationship and repeated addition or multiplication (e.g., 4 x ? = 200).

If either solution is not used, then model as another way the teacher has seen used previously.

Big Ideas

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit (2 x $\frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3}$ x 2); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also 26 x 4 = (20 + 6) x 4).

Connect

Record the solution for each of the problems:

$\frac{1}{4}$ of 40 = 10 40 ÷ 4 = 10
$\frac{1}{4}$ of 200 = 50 200 ÷ 4 = 50
$\frac{1}{4}$ of 88 = 22 88 ÷ 4 = 22

What patterns and relationships do you notice?
What is a rule for finding a fraction of a set?

Suggested Learning Outcomes

Find fractions of a set.

Generalise how to find a fraction of a set.

Independent Tasks

You have a bag of 96 lollies, and you share them equally with three friends.
What fraction do you each get?
How many lollies will you each get?

You have a bag of 123 lollies, and you share them equally with two friends.
What fraction do you each get?
How many lollies will you each get?

What is a half of 124?
What is a half of 1240?

What is a quarter of 68?
What is a quarter of 680?

What is a third of 141?
What is a third of 1410?

Curriculum Links

During Year Five
Find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., 32 of 24)

Identify, from a fractional part of a set, the whole set

During Year Six
Find a fraction or percentage of a whole number where the answer is a whole number

Identify, from a fractional part of a set, the whole set

Mathematical Language

Whole, half, halves, quarters, fourths, whole number, equivalent, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 10

Kiriwai has been given a cake to decorate. She is given 40 lollies to decorate it. She decides to split the cake into three sections and decorate each section but with a different proportion of lollies on each section.
She puts $\frac{3}{10}$ of her lollies on the first section.
She puts $\frac{2}{5}$ of the lollies on the second section.
She puts $\frac{1}{10}$ of the lollies on the third section.
How many lollies does she put on each section?
How many lollies are left over?

Kiriwai has been given a cake to decorate. She is given 90 lollies to decorate it. She decides to split the cake into three sections and decorate each section but with a different proportion of lollies on each section.
She puts $\frac{2}{9}$ of her lollies on the first section.
She puts $\frac{1}{3}$ of the lollies on the second section.
She puts $\frac{4}{9}$ of the lollies on the third section.
How many lollies does she put on each section?
How many lollies are left over?

Teacher Notes

Facilitate the students to notice that when the fraction is not a unit fraction($\frac{1}{5}$), then they have to use a multiplicative relationship to consider the sets of that fractional number

Monitor for students using vocabulary of numerator and denominator

Shareback

Select students to share who have used a representation split into fraction parts and then have used partitive division (e.g., 40 ÷ 10 = 4 and 4 x 3 = 12).

Connect

Ask students to describe how you would solve the following problems using the same solution method:

$\frac{3}{5}$ of 155

$\frac{29}{123}$ of 1369

$\frac{a}{b}$ of c

What rule can you use to find a fraction of a set?

Big Ideas

A comparison of a part to the whole can be represented using a fraction.

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b}$= a ÷ b, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3}$= 2 ÷ 3.

On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is $\frac{2}{3}$of a unit (2 x $\frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3}$x 2); each is associated with the same point on the number line..

The whole is important in naming fractions. A fraction is relative to the size of the whole or unit

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6}$ ÷ $\frac{2}{8}$ = $\frac{2}{3}$ ÷ $\frac{1}{4}$ = $\frac{2}{3}$ x $\frac{1}{4}$; also 26 x 4 = (20 + 6) x 4).

Suggested Learning Outcomes

Find fractions of a set.

Generalise how to find a fraction of a set.

Independent Tasks

Solve these equations:

$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \_ \times \_ = \_$

$3 \times \_ = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} =$

$\frac{1}{4} + ? = 2 \times \frac{1}{4}$

$\frac{1}{2} = \frac{?}{?} + \frac{?}{?} + \frac{?}{?}$

Write a story problem that would match these equations:

$\frac{4}{5} \times 6 =$

$\frac{6}{8} + \frac{2}{5} =$

$3 \div \frac{1}{2} =$

Curriculum Links

During Year Five
Find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., 32 of 24)

Identify, from a fractional part of a set, the whole set

During Year Six
Find a fraction or percentage of a whole number where the answer is a whole number

Identify, from a fractional part of a set, the whole set

Mathematical Language

Whole, thirds, ninths, tenths, fifths, equal, equivalent, fair share, partitioning, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 11

William has spent $\frac{1}{5}$ of his pocked money which is $8. How much money does he have to spend?

Louisa has planted 26 seeds which was $\frac{2}{3}$ of the packet of seeds. How many seeds does she have left?

Pat has given away $\frac{4}{12}$ of his rugby cards which is 20. How many cards did Pat begin with?

Teacher Notes

Facilitate the students to notice that when the fraction is not a unit fraction, then they have to use a multiplicative relationship to consider the sets of that fractional number

Monitor for students using vocabulary of numerator and denominator

Shareback

Find the unit fraction for the set and then use the denominator to multiply to find the whole set.

Model representing the task using a bar model of fractions and a number-line.

Connect

Ask students to describe how they would solve the following tasks:

Sophia has read $\frac{3}{7}$ of a book and is at page 75. How many pages in the book?

Carl spent $102 which was $\frac{2}{6}$ of his savings. How much did he have in his savings?

Can you come up with a rule to solve these types of tasks?

Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also $26 \times 4 = (20 + 6) \times 4$).

Suggested Learning Outcomes

Use repeated subtraction as division.

Solve problems that involve dividing a mixed number by a fraction.

Independent Tasks

Ryka has spent $\frac{1}{6}$ of his pocked money which is $9. How much money does he have to spend?

Maria has planted 25 seeds which was $\frac{5}{15}$ of the packet of seeds. How many seeds does she have left?

Lacey has given away $\frac{4}{12}$ of her rugby cards which is 20. How many rugby cards did Lacey begin with?

Curriculum Links

During Year Five

Find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., 32 of 24)

Add and subtract fractions with the same denominators including to make more than one whole.

During Year Six

Find a fraction or percentage of a whole number where the answer is a whole number

Add and subtract fractions with the same related denominators

Mathematical Language

Whole, half, halves, eighths, whole number, equal, equivalent, section, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Task 12

Solve these equations:

1. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=$

2. $\_ = 1\frac{1}{8}+\frac{1}{2}$

3. $? = \frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

4. $6\frac{5}{8}+\_ = 8$

5. $4 \times \frac{1}{4} = \_\_$

6. $\_ = 6 - 3\ \frac{4}{6}$

7. $8 \times \frac{1}{2} =$ $80 \times \frac{1}{2} =$ $800 \times \frac{1}{2} =$

8. $\frac{1}{4} - \frac{1}{12} =$

9. $\frac{6}{8}+\frac{6}{8} = 1 + \frac{?}{?}$

10. $\frac{11}{12} - \_ = \frac{2}{3}$

Be ready to explain and justify your explanations using representations and/or notation.

Teacher Notes

Facilitate the students to draw on the relationships they have identified in addition, subtraction, multiplication and division.

Notice students who are able to generalise patterns across denominators

Shareback

Select students to share who are able to draw on number properties and operational laws including the understanding of the equals sign to solve the problems.

Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into. The top number tells how many equal parts are indicated.

A fraction describes division ($\frac{a}{b} = a \div b$, a & b are integers & b 0), and it can be interpreted on the number line in two ways. For example, $\frac{2}{3} = 2 \div 3$.

On the number line, $2 \div 3$ can be interpreted as 2 segments where each is $\frac{2}{3}$ of a unit ($2 \times \frac{1}{3}$) or $\frac{1}{3}$ of 2 whole units ($\frac{1}{3} \times 2$); each is associated with the same point on the number line.

Every fraction/ratio can be represented by an infinite set of different but equivalent fractions/ratios.

Numerical expressions can be named in an infinite number of different but equivalent ways (e.g. $\frac{4}{6} \div \frac{2}{8} = \frac{2}{3} \div \frac{1}{4} = \frac{2}{3} \times \frac{1}{4}$; also $26 \times 4 = (20 + 6) \times 4$).

The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers.

Fractions with unlike denominators are renamed as equivalent fractions with like denominators to add and subtract.

Connect

Draw on student solution strategies to highlight operational laws that the students used to solve the problems (e.g., inverse relationship, commutative property).

Connect back to number and algebra unit to highlight that the properties work with all numbers including fractions.

Suggested Learning Outcomes

Solve missing number problems that involve fractions.

Solve addition problems involving fractions.

Solve subtraction problems involving fractions.

Solve multiplication problems involving fractions.

Solve division problems involving fractions.

Use operational laws to solve missing number problems that involve fractions.

Independent Tasks

Choose from one of the following Assessment tasks:

Task 1: Fractions/Proportion, Ratio and Decimals

Task 2: Fractions (set)

Curriculum Links

During Year Five

Find a fraction of a whole number, using multiplication and division facts and where the answer is a whole number (e.g., 32 of 24)

Add and subtract fractions with the same denominators including to make more than one whole.

During Year Six

Find a fraction or percentage of a whole number where the answer is a whole number

Add and subtract fractions with the same related denominators

Mathematical Language

Whole, half, halves, eighths, whole number, equal, equivalent, section, mixed numbers, numerator, denominator.

Anticipations

Solutions, Misconceptions

Assessment Task 1 - Fractions - Year 5 - 6

Put these fractions in order from smallest to biggest.

$\frac{2}{2}$ $\frac{3}{4}$ $\frac{5}{10}$ $\frac{3}{9}$ $\frac{8}{12}$ $\frac{5}{6}$

Explain and show how you know this.

Assessment Task 2 - Fractions - Year 5 - 6

Josh, Tamati and Emelia planted seeds in the garden.

Josh Planted two thirds ($\frac{2}{3}$) of a bag of 39 seeds.
Tamati planted three quarters ($\frac{3}{4}$) of a bag of 32 seeds
Emelia planted three eighths ($\frac{3}{8}$) of a bag of 48 seeds

Who planted the most seeds? Who planted the least?
Explain and show how you know this.