

A close-up photograph of several green fern fronds, showing the intricate, feathery structure of the leaves. The fronds are vibrant green and have a slightly glossy texture. They are set against a dark, blurred background, which makes the green leaves stand out. The lighting is soft, highlighting the edges and veins of the fronds.

RICH MATHEMATICAL TASK BOOKLET

NUMBER & ALGEBRA

YEAR 7- 8 ODD YEARS

Teacher Booklet

Task 1

At the Strawberry farm, there are 78 strawberry plants in each row. The Strawberry farm can fit 212 rows on their land. How many strawberry plants are there altogether?
Show your solution using two different representations.

At the Strawberry farm, there are 143 strawberry plants in each row. The Strawberry farm can fit 389 rows on their land. How many strawberry plants are there altogether?
Show your solution using two different representations.

Teacher Notes

Prior to launching the task, explain to the students that factors are the numbers that multiply to make a product with no remainder. For example, the factors of 10 are 1, 10, 5, 2. Model using a factor tree to record the factors. Ask students to work with a partner and list all of the factors for the following numbers:

36 12 49 81 25

Introduce the term, the highest common factor.

To launch the task, ask students to calculate the following problems:

$$18 \times 10 =$$

$$100 \times 25 =$$

$$40 \times 4000 =$$

Ask the students to discuss what they notice.

Be aware of students who rely on 'just add a 0' when dealing with base 10 multiplication. Explore what is happening to the numbers (getting bigger by base (10)) rather than rely on a misconception/rule.

Expect students to use equations and an area model to record solution strategies.

If students are using standard algorithm, check for procedural knowledge with understanding. The standard algorithm can be connected with the distributive property.

Shareback

Select student solution strategies that have used the distributive property, associative property or equivalence and compensation.

Use the correct mathematical language to describe these.

Distributive property

$$78 \times 212 = (70 \times 200) + (70 \times 10) + (70 \times 2) + (8 \times 200) + (8 \times 10) + (8 \times 2)$$

Associative property

$$78 \times 212 = (212 \times 10 \times 7) + (212 \times 8)$$

Equivalence and compensation

$$78 \times 212 = (80 \times 212) - (2 \times 212)$$

Big Ideas

There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.

Curriculum Links

During Year 7

Reading, writing comparing, and ordering whole numbers using powers of 10 (e.g. $10,000 = 10^4$, $1000 < 10^4$)

Using rounding and estimation to predict results and to check the reasonableness of calculations

Multiply whole numbers

During Year 8

Reading, writing comparing, and ordering whole numbers and decimals using positive and negative powers of 10

Using rounding, estimation, and benchmarks to predict results and to check the reasonableness of calculations

Multiplying and dividing whole numbers

Shareback continued

Ask students to record these as equations and model representing these using the area model. If no student solves the task using the distributive property or equivalence and compensation, then introduce either solution strategy as an alternative model previously used by other students.



Connect

Ask students to describe how the following equation could be solved using the distributive property and equivalence and compensation:

$$164 \times 56 =$$

Model to students how this connects to a vertical place-value algorithm using place value language.

Suggested Learning Outcomes

Explain and justify the use of the distributive property in multiplication.

Explain and justify the use of the associative property in multiplication.

Explain and justify the use of equivalence and compensation in multiplication.

Represent reasoning using different forms of notation including equations and an area model.

Independent Tasks

Solve the following equations:

$$194 \times 55 =$$

$$176 \times 42 =$$

$$131 \times 329 =$$

$$215 \times 197 =$$

Explain what patterns you used to help solve the equations.
Would the patterns always work?

Mathematical Language

Distributive property, associative property, area, equivalence, compensation, factor, product

Anticipations

Solutions, Misconceptions

Task 2

The Lottery Foundation has \$3818 available for funding for sports team. They have 53 sports team that apply. How much money will each team receive?

What numbers (above a thousand) could you start with, that would mean that each team only receives dollars and no cents?

Teacher Notes

Prior to launching the task, teach the students to play the divisibility game using a set of 0 – 9-digit cards. In pairs ask the students choose and place a card to the right of the existing card. The rules are ... after two cards are placed, the two-digit number must be divisible by 2; after three cards are placed, the three-digit number must be divisible by 3; after four cards are placed, the four-digit number must be divisible by 4, and so on. Continue playing until one cannot place a card. Ask students to discuss what they notice about strategies that are helpful.

Notice students who are using addition or subtraction and support them to re-work as multiplicative thinking.

Notice students who use doubling and support them to recognise this as multiplying by two. Press students to use larger factors such as 5 or 10.

Shareback

Select student solution strategies where they have used the inverse relationship of multiplication and division or the partial quotient/distributive property (or a mixture of both) in the solution. If either solution strategy has not been used, introduce this as a solution strategy that students have used previously.

Inverse relationship

$$3818 \div 53 = ?$$

$$53 \times ? = 3818$$

$$53 \times 10 = 530 \dots$$

Partial quotient/Distributive property

$$3818 \div 53 = (1060 \div 53) + (1060 \div 53) + (1060 \div 53) + (530 \div 56) + (108 \div 53)$$

Big Ideas

There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.

Curriculum Links

During Year 7

Multiplying whole numbers

Using divisibility rules to identify numbers that are divisible by 2, 3, 4, 5, 6, 8, 9, and 10

Dividing whole numbers by one- or two-digit divisors (e.g. $327 \div 5 = 65.4$ or $652 \frac{2}{5}$)

During Year 8

Multiplying and dividing whole numbers (e.g. $327 \div 15 = 21.8$ or $214 \frac{2}{5}$)

Connect

Ask students to describe how the following equation could be solved using the distributive property and equivalence and compensation:

Ask students to describe how you would solve the following equation using the inverse relationship and the partial quotient/distributive property:

$$6626 \div 28 =$$

Suggested Learning Outcomes

Explain and justify the use of the partial quotients/distributive property in division.

Explain and represent the inverse relationship of multiplication and division.

Represent reasoning using different forms of notation.

Independent Tasks

Have a go at solving the following tasks involving exponents.

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

$$4^5$$

$$8^4$$

$$5^6$$

$$7^3$$

$$6^8$$

Check your answer with a calculator.

What patterns do you notice?

Mathematical Language

Distributive property, inverse relationship, equivalence, factor, product, quotient, divisor, dividend.

Anticipations

Solutions, Misconceptions

Task 3

Pascal sweet factory put 304 sweet packets in each large container ready to be sent for packaging. Each fifteen minutes the machine sorts 6806 packets of sweets. How many containers would be used every fifteen minutes and many packets of sweets would be left over?

For what numbers would there be no packets of sweets left over but almost the same number of containers used?

Teacher Notes

Prior to launching the task, ask the students to play the divisibility game again (see teacher notes in Task 2 for the rules), this time with the aim of creating the longest number possible. Ask students to discuss what they notice about strategies that are helpful.

Notice students who are using addition or subtraction and support them to re-work as multiplicative thinking.

Notice students who use the inverse property or who are using partial quotients/distributive property in their calculations.

Shareback

Select student solution strategies where they have used the partial quotient/distributive property in the solution.

Connect

Ask students to describe how you would solve the following equation using partial quotients/distributive property:

$$6206 \div 304 =$$

Use modelling to show connections between the use of the partial quotients/distributive property and the standard division algorithm.

Big Ideas

There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.

Curriculum Links

During Year 7

Reading, writing comparing, and ordering whole numbers using powers of 10 (e.g. $10,000 = 10^4$, $1000 < 10^4$)

Multiplying whole numbers

Using divisibility rules to identify numbers that are divisible by 2, 3, 4, 5, 6, 8, 9, and 10

Dividing whole numbers by one- or two-digit divisors (e.g. $327 \div 5 = 65.4$ or $652 \frac{2}{5}$)

During Year 8

Reading, writing comparing, and ordering whole numbers and decimals using positive and negative powers of 10

Multiplying and dividing whole numbers (e.g. $327 \div 15 = 21.8$ or $214 \frac{2}{5}$)

Suggested Learning Outcomes

Explain and justify the use of the partial quotients/distributive property in division.

Explain and represent the inverse relationship of multiplication and division.

Represent reasoning using different forms of notation.

Mathematical Language

Distributive property, inverse relationship, equivalence, factor, product, quotient, divisor, dividend.

Independent Tasks

Solve the following equations:

$$678 \div 25 =$$

$$8575 \div 405 =$$

$$6344 \div 28 =$$

$$9333 \div 322 =$$

Anticipations

Solutions, Misconceptions

Task 4

Abundant numbers are numbers which are greater than the sum of its factors (without itself).

Work in your group to see whether 10 is an abundant number.

Is 48 an abundant number?

Work together to find as many abundant numbers as you can between 0 - 100.

Discuss the patterns that you could use to help you with the task.

Develop a range of conjectures related to abundant numbers and see whether you can prove them.

Teacher Notes

Prior to launching the task, ask the students to find the highest common factor of:

- 16 and 12
- 12 and 30
- 60 and 96
- 36 and 84

Provide students with a 100 square to support their exploration. Facilitate students to identify the patterns that they notice and describe and test these.

Abundant numbers are numbers which are greater than the sum of its factors (without itself). Deficient numbers are numbers that are less than the number of its factors (without itself). Perfect numbers are numbers that equal the sum of their factors (without itself).

For example:

- 12 is an abundant number because its divisors (excluding itself) are 1, 2, 3, 4, 6, and their sum is 16, which is greater than 12.
- 8 is a deficient number because its divisors (excluding itself) are 1, 2, and 4, and their sum is 7, which is less than 8.

For the independent task, provide students with a 100 square to record the patterns.

Big Ideas

Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways.

Curriculum Links

During Year 7

Using exponents and identifying square roots for square numbers up to at least 144

Finding the highest common factor (HCF) of two numbers under 100, and finding the least common multiple (LCM) of two numbers under 10

Using divisibility rules to identify numbers that are divisible by 2, 3, 4, 5, 6, 8, 9, and 10

During Year 8

Whole numbers greater than zero are either prime, composite, or the number 1.

- *A prime number has exactly two distinct factors: 1 and the number itself.*
- *A composite number has more than two distinct factors.*
- *1 is neither prime nor composite.*

Shareback

Select and sequence student solutions which identify patterns and ask students to explain and justify these.

All abundant numbers are even.
Abundant numbers have six factors or fewer.

Ask students to investigate the conjectures about abundant numbers and prove or disprove these.

Connect

Use a 100s board and colour in all the abundant numbers. Ask students to make conjectures about further patterns they notice.

Can a prime number be an abundant number?

Suggested Learning Outcomes

Identify that factors are the numbers that multiply to make a product with no remainder.

Calculate factors for different numbers.

Explain and justify patterns and relationships in factors and abundant numbers.

Independent Tasks

People throughout history have always looked for patterns in numbers.

Mathematicians noticed that some numbers are equal to the sum of all of their factors (but not including the number itself). These are called perfect numbers.

Another pattern is prime numbers which can only be divided by itself and by 1 without remainders.

On the 100 square use different colours to mark the following: perfect numbers; prime numbers; abundant numbers.

What patterns do you notice?

Mathematical Language

Factor, multiple, abundant numbers, product, digit, conjecture.

Anticipations

Solutions, Misconceptions

Task 5

Can you work together in your group to solve these number sentences? Make sure that you develop an explanation and justification.

$$167 + 48 = 169 + \underline{\hspace{1cm}}$$

$$153 - 86 = \underline{\hspace{1cm}} - 76$$

$$545 + 78 - \underline{\hspace{1cm}} = 543$$

$$\underline{\hspace{1cm}} \times 14 = 32 \times 7$$

$$72 \div 12 = (48 \div 12) + (\underline{\hspace{1cm}} \div 12)$$

Teacher Notes

Before you launch the task, ask the students to discuss these true and false number sentences and justify their thinking. $547 + 368 = 549 + 36$

$$487 + 8 + 7 = 487 + 16$$

$$63 - 47 = 61 - 45$$

$$387 = 385$$

$$7 \times 9 = (5 \times 9) + 9$$

$$8 \times 6 = (10 \times 6) - 6 - 6$$

Ensure that students understand what true and false means. Introduce notation of not equal ($\neq$) for the number sentences that they think are false.

Students may initially treat the equals sign as an operator or indication to write the answer next.

Students also may compute each side to work out whether they are equal. Notice students who use the relationships across the equals sign to see whether there is balance.

Highlight to the students to look across the equals sign and find the relationships between numbers to the left and the numbers on the right. Notice students who use the relationships across the equals sign to see whether there is balance.

Highlight the students relational responses (e.g., noticing the $+2 - 2$ relationships).

Press for use of arrows and notations to highlight the relationships.

The image shows two handwritten equations with arrows indicating relationships between numbers on either side of the equals sign.

Equation 1: $34 - 17 = 35 - 18$. An arrow labeled $+1$ points from 34 to 35. Another arrow labeled $+1$ points from 17 to 18.

Equation 2: $116 - 89 = 126 - 99$. An arrow labeled -10 points from 116 to 126. Another arrow labeled -10 points from 89 to 99.

Big Ideas

Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.

Curriculum Links

During Year 7

Forming and solving one- and two-step linear equations with integer solutions (e.g. $t + 7 = 12$, $5s + 3 = 18$)

During Year 8

Forming and solving linear equations with rational solutions (e.g. $t + 7 = 6.5$, $5s + 9 = -18$)

Shareback

Select student solution strategies that use relational reasoning.

Connect

Is the number that goes in the __, the same number in both of these equations?

$$4 \times _ + 12 = 24$$

$$4 \times _ + 12 - 6 = 24 - 6$$

Explain why or why not. Can you make a conjecture about this relationship?

Suggested Learning Outcomes

Explain and justify relationships between numbers in an equation.

Write statements of equivalence in words and using notation.

Solve equivalence problems and explain and justify the solutions.

Independent Tasks

Work out which number sentences are true or false and explain your reasoning.

$$566 + 388 = 564 + 386$$

$$73 - 38 = 71 - 36$$

$$288 + 16 = 288 + 8 + 9$$

$$53 - 27 = 63 - 17$$

$$385 = 385$$

$$6 \times 12 = (6 \times 10) + 6 + 6$$

$$9 \times 7 = (10 \times 7) - 7 - 7$$

$$8 + 9 + 10 = 11 + 12 + 13 + 14$$

Find the missing numbers:

$$58 + 37 = _ + 39$$

$$_ + 436 = 579 + 426$$

$$512 - 269 = 412 - _$$

$$_ - 346 = 621 - 348$$

$$15 \times 38 = 38 \times 5 \times _$$

$$378 \div 18 = 378 \div _ \div 3 \div 3$$

Mathematical Language

Equivalent, equal sign.

Anticipations

Solutions, Misconceptions

Task 6

What are the possible values for $b + b = 14$?

What are possible values for $y + g = 12$?

Now, work together in your group to solve these equations and justify your solution. Make sure that everyone can explain and justify your responses.

$$y - 12 = 8$$

$$15 - c + 4 - c = 9$$

$$y \times 4 + y - y + 3 = 27$$

$$8g + 7 = 39$$

$$12b - 23 = 49$$

Teacher Notes

Before you launch the problem, ask the students to work with a partner and write an expression to match these situations:

1. I have some stickers and I give seven away.
2. I have some stickers and I get four more.
3. I have some stickers and I get three more and then I get four more.
4. I have some stickers and I get six more and then I triple the total amount of stickers I have.

Introduce each one and share student solutions that use a variable to model the equation. If all students put numbers, then problematise this by asking, do we know how many? If we don't know, we can use a letter to represent any number.

Introduce to students that $2n = 2 \times n$ as notation.

Launch the first part of the problem and then bring the students back to share ideas. Highlight that b will be the same number so only one solution. Address potential misconception that y and g cannot both equal 6. Highlight that $y + g$ has multiple solutions including $y = 6$ $g = 6$

Values of variables: Variables can have any values - a letter is assigned to this value. The same variable has the same value in an equation. Different variables can have the same value.

Discuss and explore with the students that equations remain balanced as long as you use the properties of equality. An operation conducted on one side of the equal sign must be applied on the other. Apply the inverse of an operation to cancel it out or remove it. The goal in solving linear equations is to isolate the unknown variable by applying the inverse to remove other known variables.

Big Ideas

Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.

Curriculum Links

During Year 7

Form and solve one-step linear equations (e.g., $t + 7 = 12$, $2s = 14$)

During Year 8

Form and solve one- or two-step linear equations (e.g., $5s + 3 = 18$)

Shareback

Select student solution strategies that use inverse relationships and the properties of equality.

Connect

The solution to $5k + 23 = 68$ is $n = 9$

What is the solution to $5k + 23 + 14 = 68 + 14$?

What conjecture can you make from this?

Conjecture could be represented as:

If $a + b = c$ then $a + b - d = c - d$

or

If $a + b = c$ then $a + b + d = c + d$

Suggested Learning Outcomes

Explain and justify how when the value of one variable is known the value of the other variable can be found by solving the equation.

Use inverse relationships and understanding of properties of equality to solve equations.

Independent Tasks

Solve the following equations:

$$7a = 49$$

$$y - 14 = 8$$

$$19 = p - 4$$

$$32 \div m = 8$$

$$5f + 6 = 31$$

$$11r - 18 = 48$$

$$3q + 7 = 25$$

$$9d - 5 = 76$$

Mathematical Language

Unknown, variable, inverse relationships, equivalence, equation, values

Anticipations

Solutions, Misconceptions

Task 7

Work together in your group to solve these equations and justify your solution. Make sure that everyone can explain and justify your responses.

$$8m = 3m + 25$$

$$4 \times h + 7 \times h = 40 + 26$$

$$24 = 4v - 16 + v$$

$$6n + 5 = 29 - n + 3n$$

$$4j - 6 = 2j + 4$$

Teacher Notes

Before you launch the task, revisit the conjectures made in the previous lesson that established that equations remain balanced as long as you do the same thing to both sides.

Ask the students:

Malia said “ $t + 3$ is always less than $5 + t$ ”
Is this always, sometimes, or never true?

Discuss and explore with the students that equations will remain balanced as long as you use the properties of equality. An operation conducted on one side of the equal sign must be applied on the other. Apply the inverse of an operation to cancel it out or remove it. The goal in solving linear equations is to isolate the unknown variable by applying the inverse to remove other known variables.

Shareback

Select student solution strategies that use inverse relationships and the properties of equality to isolate and solve for the unknown.

Big Ideas

Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.

A solution to an equation is a value of the unknown or unknowns that makes the equation true. Properties of equality and the inverse property can be used to generate equivalent equations and find solutions.

Curriculum Links

During Year 7

Form and solve one-step linear equations (e.g., $t + 7 = 12$, $2s = 14$)

During Year 8

Form and solve one- or two-step linear equations (e.g., $5s + 3 = 18$)

Connect

Remind students of BEDMAS and ask students to describe the steps that you could take to simplify the following:

$$5(b + 4) + 3 =$$

$$10 + 3(h + 3) =$$

Then ask them how they would solve:

$$2(h + 2) + 10 = 24$$

Suggested Learning Outcomes

Explain and justify how when the value of one variable is known the value of the other variable can be found by solving the equation.

Use inverse relationships and understanding of properties of equality to solve equations.

Independent Tasks

Solve the following equations:

$$13 + r = 30$$

$$k - 8 = 14$$

$$7d = 42$$

$$6b + 5 = 23$$

$$4k - 3 = 17$$

$$42 + 5t = 8t$$

$$7d + 4 = 2d + 29$$

$$7k - 13 = 2y + 12$$

Mathematical Language

Unknown, variable, inverse relationships, equivalence, equation, values.

Anticipations

Solutions, Misconceptions

Task 8

In your groups look at the equations and develop a story that matches the equation. Make sure that everyone in your group can explain and justify why the story matches the equation. Have a go at solving the story problems that you have created:

$$-20 + 5 = \underline{\quad}$$

$$\underline{\quad} - -3 = 6$$

$$-7 + \underline{\quad} = -11$$

Teacher Notes

Before you launch the task, ask students to brainstorm everything that they know about negative numbers and record their ideas.

Integers are an extension of whole numbers which include positive and negative whole numbers that are opposites (...-2, -1, 0, 1, 2,...).

Possible story contexts: height above sea level, scoring in sports/games e.g. golf, Bridge, positive/negative spaces e.g. digging holes, lifts which go below ground, temperatures below zero.

Shareback

Select student solution strategies that use inverse relationships and the properties of equality to isolate and solve for the unknown.

Connect

Ask students to compare the stories and notice similarities and differences.

Big Ideas

Mathematical situations can be represented as equations which include both positive and negative integers.

A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.

Curriculum Links

During Year 7

Locating integers on a number line

Representing addition and subtraction of integers using a number line

During Year 8

Locating negative and positive numbers on a number line

Comparing and ordering negative and positive numbers using a number line (e.g. $-3.4 < -3$)

Suggested Learning Outcomes

Represent in a mathematical story a situation in a problem which has both negative and positive numbers.

Explain and justify how integers are an extension of whole numbers and include both positive and negative whole numbers.

Mathematical Language

Integers, negative number, positive number.

Independent Tasks

Look at the equations and develop one or more stories that match each equation.

$$-3 + -1 =$$

$$-15 - \underline{\quad} = 11$$

$$\underline{\quad} + 24 = 0$$

$$-15 - 18 =$$

Anticipations

Solutions, Misconceptions

Task 9

In your groups represent your reasoning on a number line to show how you solved each of these problems:

$$-7 + 2 =$$

$$-19 - -11 =$$

$$4 - -9 =$$

$$-12 + 8 =$$

$$6 - 15 =$$

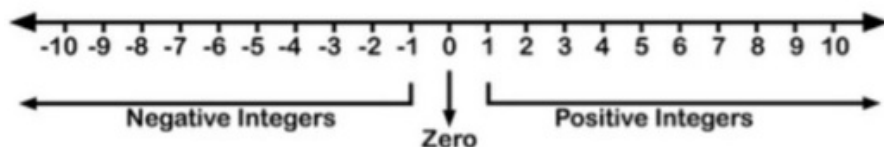
$$-12 + -4 =$$

Teacher Notes

Before you launch the task, ask students to share when they see negative numbers in life?

[Lifts, mortgages, temperature]

Model an empty number line on the board and ask students to discuss where the numbers would go if you were counting from negative 5 to positive 5. Use the number line to represent the location/relationship of negative/positive numbers to each other.



Highlight that two integers that are the same distance from the origin in opposite directions are called opposites and when added cancel each other making 0.

Highlight difference between the use of - as an operation symbol (subtraction) and direction symbol (direction/size of movement) for negative numbers.

Consider using physical materials to represent positive and negative numbers e.g. black counters (positive) and red counters (negative).

Shareback

Select student solution strategies that will support a discussion of the relationship between addition and subtraction when working with integers.

Big Ideas

Mathematical situations can be represented as equations which include both positive and negative integers.

A real quantity having a value less than zero is negative. Positive and negative numbers are opposites.

Curriculum Links

During Year 7

Locating integers on a number line

Representing addition and subtraction of integers using a number line

During Year 8

Locating negative and positive numbers on a number line

Comparing and ordering negative and positive numbers using a number line (e.g. $-3.4 < -3$)

Connect

What patterns did you notice when you were adding and subtracting positive and negative numbers?

Suggested Learning Outcomes

Solve simple addition and subtraction equations using integers.

Use a number line to represent the relationship between positive and negative integers in equations.

Explain and justify the role of zero as neither positive nor negative.

Explain and justify the use of - as an operation symbol (subtraction) and direction symbol (direction/size of movement) for negative numbers.

Mathematical Language

Integers, negative number, positive number.

Independent Tasks

Solve these equations (use an empty line if it helps):

$$17 + -9 =$$

$$-16 - -24 =$$

$$-36 + -16 =$$

$$21 - -43 =$$

$$-265 - 78 =$$

$$-273 + -168 =$$

$$-144 - -223 =$$

Anticipations

Solutions, Misconceptions

Task 10

Tiare is solving a division problem that her teacher gave her.

She is solving this: $352 \div 16 =$

Tiare solves it by writing $352 \div 16 = (160 \div 16) + (160 \div 16) + (32 \div 16)$

Do you agree with Tiare's solution? In your group, develop an explanation of why this works or why you think it doesn't work.

Can you develop examples with other numbers which also use this pattern?

Does this pattern work with multiplication?

Teacher Notes

Facilitate students to focus on the solution strategy and generalisation rather than calculating the solution.

Notice and highlight the conjectures that students develop.

Shareback

Select students who use the relationship and properties rather than calculating.

Highlight to the students that you do not need to calculate but can use the relationship to solve different equations.

Connect

Taine thought that Tiare should have solved the problem like this:

$$352 \div 16 = 352 \div 2 \div 4 \div 2$$

Do you agree with Taine?

Which way works and why?

How would Taine solve the following problem $528 \div 24$?

Big Ideas

There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties. Addition and subtraction and multiplication and division have an inverse relationship.

Curriculum Links

During Year 7

Using rounding and estimation to predict results and to check the reasonableness of calculations

Multiply whole numbers
Dividing whole numbers by one- or two-digit divisors (e.g. $327 \div 5 = 65.4$ or $652 \frac{2}{5}$)

Using divisibility rules to identify numbers that are divisible by 2, 3, 4, 5, 6, 8, 9, and 10

Evaluating expressions using the order of operations

During Year 8

Using rounding, estimation, and benchmarks to predict results and to check the reasonableness of calculations

Evaluating expressions with integers, using the order of operations
Multiplying and dividing whole numbers (e.g. $327 \div 15 = 21.8$ or $214 \frac{4}{5}$)

Suggested Learning Outcomes

Explain and justify how you can solve division problems by splitting the divisor into factors.

Represent that the equals sign as a statement of balance and show which operations to both sides of an equation preserve that balance.

Mathematical Language

Factors, divisor, dividend, associative property.

Independent Tasks

Find the missing numbers:

$$54 \times _ = (54 \times 5) + (54 \times 10) + (54 \times 3)$$

$$288 \div 3 \div 2 \div _ = 288 \div 12$$

$$38 \times 42 = 38 \times 3 \times _ \times 7$$

$$_ \div 24 = (240 \div 24) + (48 \div 2) + (48 \div 2)$$

Anticipations

Solutions, Misconceptions

Task 11

Choose an item of clothing to buy (e.g., a pair of jeans). Use the internet to find the cost of comparable items from different stores or brands.

Estimate how many times you might wear this item. Now calculate the cost per wear, depending on where you choose to buy.

Illustrate the results using a table or spreadsheet.

For each item, write a sentence about the quality of the item and how long you predict it will last in terms of both wear and tear and fashion trends.

How and why did the cost per wear vary between different items of clothing?

Teacher Notes

Facilitate students to find at least a minimum of four options for the clothing item including both low-cost and higher cost items and factors on the website related to sustainability.

Support students to consider factors which may impact the lifespan and how long they would typically keep and wear an item of clothing. This could include sizing, quality of fabric, style, etc. All of these aspects are important considerations in making financial choices rather than just costs.

Using month measurement will make it easier for students to use a ratio for the cost per wear, for example, assume the number of wears per month, and also the lifespan in months. Ensure that students estimate how many times the piece of clothing will be worn per month and then a cost per wear can be developed through using ratio.

Students should present their findings using a table or excel.

Shareback

Select students who have used a table or spreadsheet to compare the costs including the expected lifespan and cost per wear.

Connect

What is important and what factors do you and your family consider when choosing clothing to buy?

Big Ideas

Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.

Patterns and relationships can be used, represented, and generalised in a variety of ways.

Curriculum Links

During Year 7

Calculating the total cost and change for a transaction involving any amount of money

During Year 8

Creating and comparing weekly, monthly, and yearly finance plans (e.g. for saving plans, phone plans, budgets, and 'buy now, pay later' services)

Suggested Learning Outcomes

Investigate, explain and justify the best buy considering personal preferences.

Create a financial plan using a spreadsheet or table.

Use a ratio to work out the cost per wear of an item of clothing.

Mathematical Language

Cost, best buy, ratio, dollars, cents.

Independent Tasks

Choose a pair of shoes to buy. Use the internet to find the cost of comparable items from different stores or brands.

Estimate how many times you might wear this item. Now calculate the cost per wear, depending on where you choose to buy.

Illustrate the results using a table or spreadsheet.

For each item, write a sentence about the quality of the item and how long you predict it will last in terms of both wear and tear and fashion trends. How and why did the cost per wear vary between different brands of shoes?

Anticipations

Solutions, Misconceptions

Task 12

Look at the comparison table below.

	Amazon Prime	Neon	Disney Plus	Netflix
Details	Enjoy exclusive Amazon Originals as well as popular movies and TV shows	Binge the best. Huge range of TV shows and movies handpicked for Kiwis by Kiwis.	Disney Plus. Something for everyone!	Unlimited TV shows, movies and more.
Special offer	7-day free trial	Annual payment of \$199.99	Annual payment of \$149.99 for standard plan Annual payment of \$189.99 for premium plan.	
Pricing	\$10.99 per month	\$12.99 per month (basic plan) \$19.99 per month (standard plan)	\$14.99 per month (standard plan) \$18.99 per month (premium plan)	\$14.99 per month (basic plan) \$20.99 per month (standard plan) \$27.99 per month (premium plan)
Fine print	Cancel anytime	Cancel before end of billing period. No refunds.	Cancel before end of billing period. No refunds.	Cancel anytime

Use a spreadsheet or table to compare the differing options in relation to monthly and yearly plan costs.

Make at least 5 mathematical statements comparing the pricing of these streaming services.

Use appropriate mathematical and financial language.

Teacher Notes

Have calculators available for the students to use for the task.

Expect students to use a spreadsheet (Excel if possible) to map out the monthly costs and use this to find out the yearly costs.

Students should be facilitated and supported to compare the costs before between the monthly and annual plans for the same company and across different companies. For example, the annual payment for Neon is two months less than paying monthly which is equivalent to around 16% discount.

Big Ideas

Equations show relationships of equality between parts on either side of the equal sign. The properties of equality are: If the same real number is added or subtracted to both sides of an equation, equality is maintained; If both sides of an equation are multiplied or divided by the same real number (not dividing by 0), equality is maintained; Two quantities equal to the same third quantity are equal to each other.

Patterns and relationships can be used, represented, and generalised in a variety of ways.

Curriculum Links

During Year 7

Calculating the total cost and change for a transaction involving any amount of money

During Year 8

Creating and comparing weekly, monthly, and yearly finance plans (e.g. for saving plans, phone plans, budgets, and 'buy now, pay later' services)

Shareback

Select students who have used a table or spreadsheet to compare the costs including the cost of different monthly or yearly plans and who have made statements with evidence from the costing plans.

Connect

If Netflix had a special offer for two months free, how would that impact their overall pricing in relation to the other options?

Suggested Learning Outcomes

Investigate, explain and discuss different costings and options for similar services.

Create a financial plan using a spreadsheet or table.

Use percentages to make comparisons between costs.

Mathematical Language

Cost, best buy, percentage, dollars, cents, discount.

Independent Tasks

Select One of the following assessment tasks as the independent activity.

Task 1 - Multiplication and Division

Task 2 - Multiplication and Division

Task 3 - Properties of Numbers and Operations

Task 4 - Properties of Numbers and Operations

Task 5 - Integers and Negative Numbers

Task 6 - Integers and Negative Numbers

Anticipations

Solutions, Misconceptions

Assessment Task 1 - Number and Algebra - Year 7-8

At the event centre, there are 225 rows of seats. Each row has 179 seats in it. How many seats are there altogether?

The library is moving. They have 3248 books and can fit 76 books into each box. How many boxes will be needed for the books?

Write your own multiplication or division problems. Show how you would solve them.

Assessment Task 2 - Number and Algebra - Year 7-8

At the shopping mall, there are 124 rows for carpark. Each row has spaces for 288 cars. How many carpark are there altogether?

A school is going on a trip and there are 1642 people who need transport. Each bus can take 38 people. How many buses will be needed?

Write your own multiplication or division problems. Show how you would solve them.

Assessment Task 3 - Number and Algebra - Year 7-8

$$76 \times 15$$

$$37 + 43 + 40 + 36$$

$$99 \div 3 \div 3$$

$$7 \times 86$$

$$4 \times 66$$

$$6^3$$

$$99 \div 9$$

$$(70 \times 5) + (70 \times 10) + (6 \times 10) + (6 \times 5)$$

$$12 \times 22$$

$$37 + 40 + 36 + 43$$

$$6 \times 6 \times 6$$

$$(7 \times 90) - (7 \times 4)$$

Look at the number sentences above.
Describe what patterns you can find.
Why do your patterns work?

Do they work with other numbers?
Will they always work? Explain and justify your thinking

Assessment Task 4 - Number and Algebra - Year 7-8

$$84 \times 16$$

$$56 + 79 + 44 + 38$$

$$88 \div 2 \div 2$$

$$6 \times 47$$

$$3 \times 99$$

$$54$$

$$88 \div 4$$

$$(80 \times 10) + (80 \times 6) + (4 \times 10) + (4 \times 6)$$

$$9 \times 33$$

$$38 + 56 + 79 + 44$$

$$5 \times 5 \times 5 \times 5$$

$$(6 \times 50) - (6 \times 3)$$

Look at the number sentences above.

Describe what patterns you can find.

Why do your patterns work?

Do they work with other numbers?

Will they always work? Explain and justify your thinking

Assessment Task 5 - Number and Algebra - Year 7-8

Why do we need negative numbers?

Give examples of how negative numbers can be useful.

Assessment Task 6 - Number and Algebra - Year 7-8

Write a story that matches the equation.
Solve the story problem.

$$-16 + 20 =$$

$$15 - \underline{\quad} = 18$$

$$-12 + -6 =$$

$$\underline{\quad} - -9 = -11$$

$$-7 + \underline{\quad} = 10$$

$$-2 - 9 =$$