DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

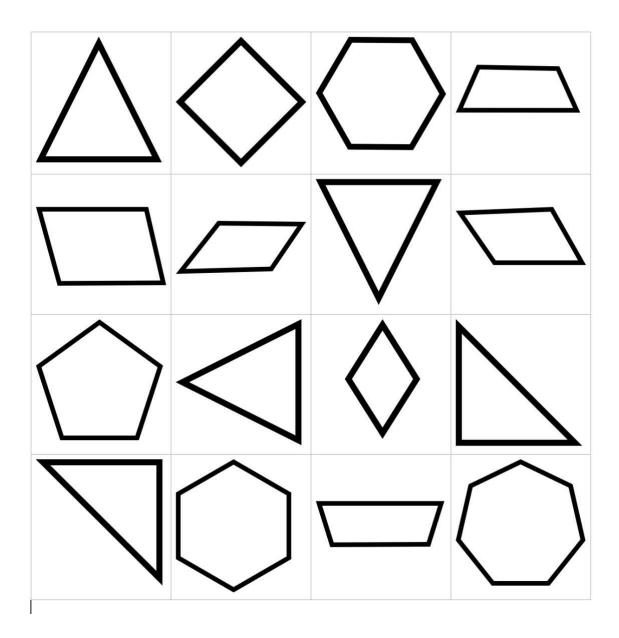
Geometry – Shape and Space Level 4 (Year 7/8) Copy Masters

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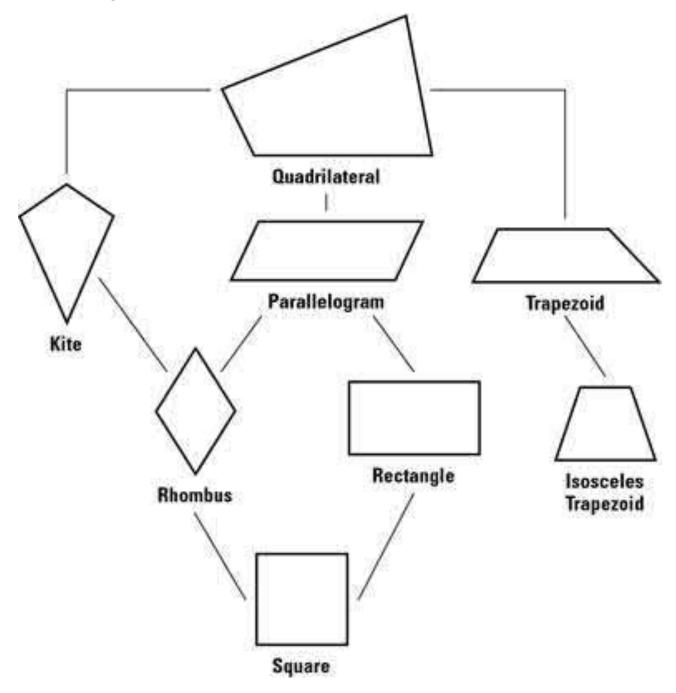
- 1. Can you sort these shapes into different groups? As you sort them, talk with your group about what you notice about them. What properties do they have that are the same? Different?
- 2. Randomly place a shape in the middle. Take turns to find other shapes which have properties the same as the first shape.

The rule is that you have to name the properties of each new shape as it is added, and the properties that match the first shape. Do this again, starting with a different shape.

Task 1 Resource







Task 1 (independent)

- 1. Make 2 squares with your sticks. How many sticks did you need?
- 2. Make a rectangle with the sticks which is made up of 2 squares joined together.
- 3. Make 4 squares with your sticks. How many sticks did you need?
- 4. Make a 2 by 2 large square with the sticks which is made of 4 squares joined together to make one large square. How many sticks did you need this time? Why do you need less?
- 5. On your paper draw a rectangle made of 2 squares.
- 6. On your paper draw a 2 by 2 large square made up of the 4 smaller squares.

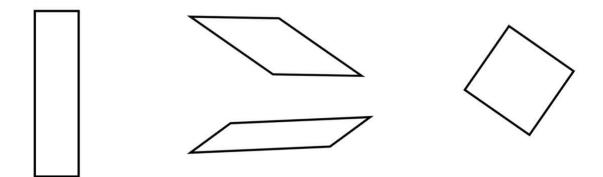
Clever experts are you ready to develop lists of properties for quadrilaterals.

Your job is to look at all the shapes on your sheet and list as many properties as you can that apply to all of the shapes on the sheet.

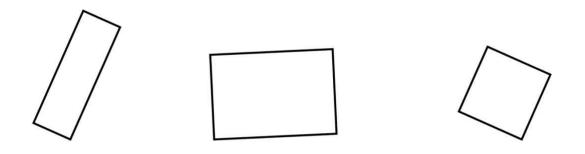
Make sure that you use tools to check such things as angles, side lengths, angle congruence, and line symmetry.

Hint: Use the terms 'at least...', 'only...', 'at most...', and 'because...'.

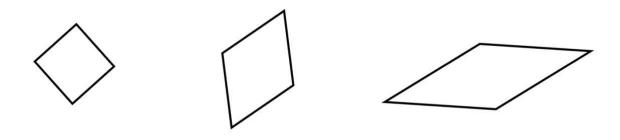
Task 2 – Page 1



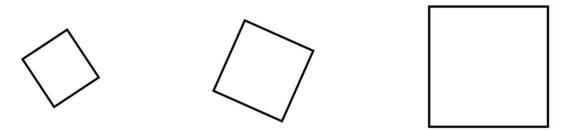
Task 2 – Page 2



Task 2 - Page 3



Task 2 – Page 4



Task 2 (independent)

- Draw what you think a 4 by 3 shape looks like which is made of 12 squares which are all the same. Check whether you are right. If you need to, keep drawing it until you are right.
- 2. Use the grid and/or dotty paper to draw the 4 by 1 shape, the 4 by 2 shape and the 4 by 3 shape. Can you make these larger and smaller?

Use the dotty and/or squared paper to draw squares and rectangles which are made up of many different smaller squares. Record what you notice about the lines.

Ready again to be property sorter? We are going to look at polygons. The word polygon is from Greek, poly means many. So, you have to be ready to be able to sort by properties across cases of shapes which are all polygons!

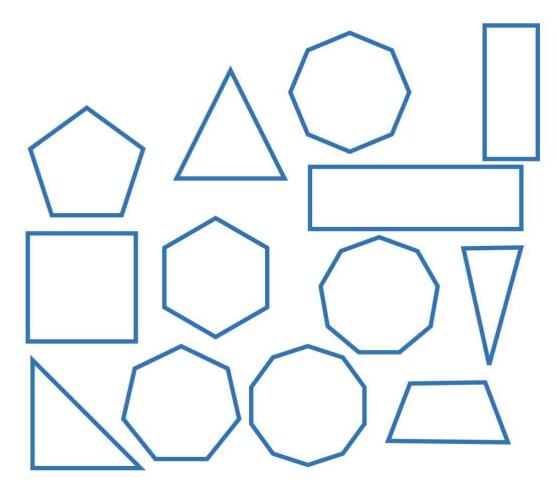
Here you have a set of polygons all mixed up. With your group can you sort these polygons into different groups by their properties.

What do you notice about their properties? Can you come up with a list of attributes you have decided are shared by **all** the polygons in each set?

As a property sorter be ready to explain and justify your list of attributes shared by the polygons in each set.

What about across the whole set of polygons? Can you make up **if** statements about the whole set of polygons?

Task 3 – Polygons



Task 3 (independent)

Did you know that mathematics and art are closely related?

Use 2-dimensional shapes to design and make a drawing.

Describe your drawing using the properties of shapes.

For example: In my drawing of a cat, I have used two circles for the eyes. I inserted two rectangles for the pupils of each eye. I used a hexagon for the main body of the cat. This hexagon has 6 sides and 2 right-angles...etc.

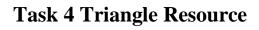
Today as a property sorter you are going to sort a special sort of polygons.

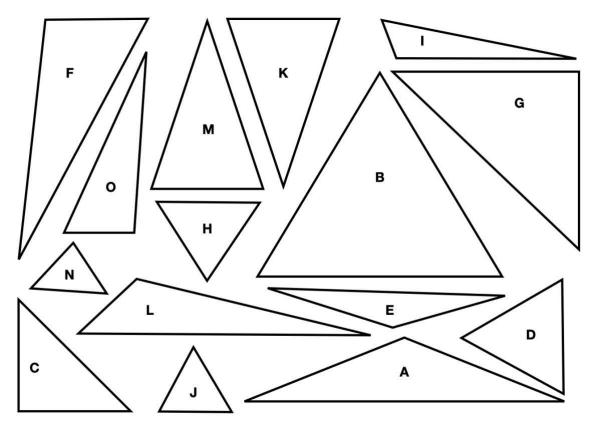
Your challenge is to sort the whole set into three groups. But wait! There is an important rule you need to follow as you do this. **No triangle is allowed to belong to two groups.**

When you have sorted them into three groups record the properties of each group.

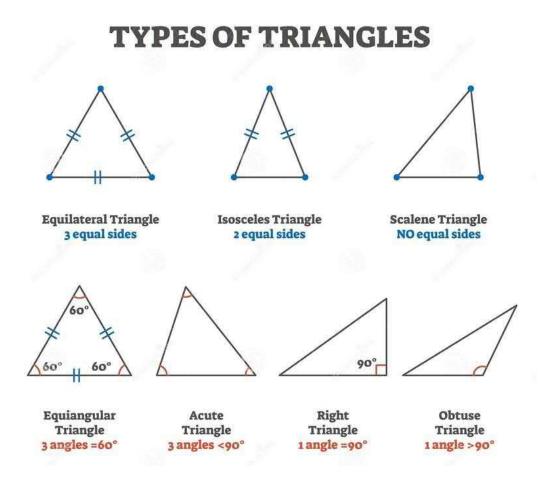
Now start again. Re-sort the set into another three groups which are different from your first set. Record the properties of this new group.

Start to develop some **if** statements.



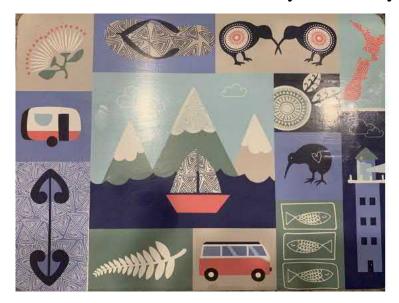


Task 4 – Supporting Resource



Task 4 (independent)

What do you notice about all the shapes on the wrapping paper that are the same? That are different? Look closely. What do you notice?



Sort the shapes into classes and sub classes according to geometrical properties. Record your explanation and justification.

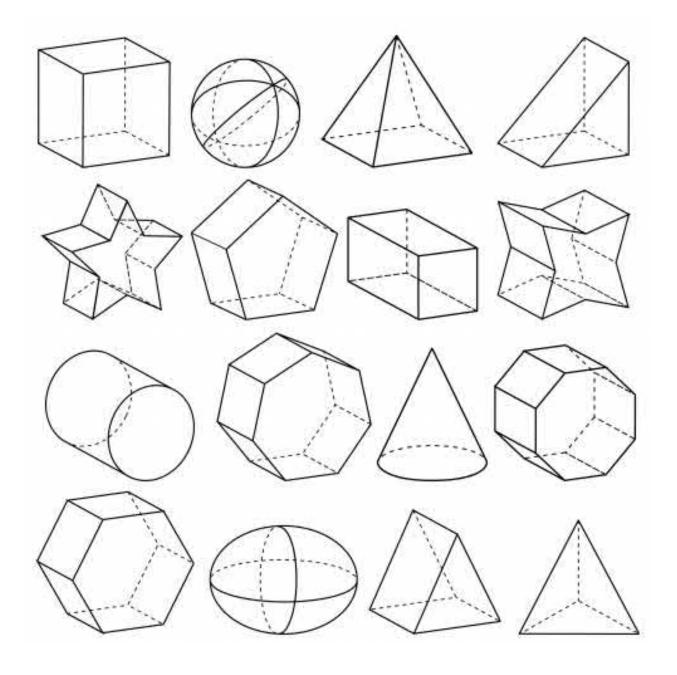
Talk with your buddy about what you notice about the shape of these different things.

Can you sort them into groups which you think are the same?

Can you sort them into groups which you think are different?

Be ready to explain and justify why you sorted them into the different groups.

Task 5 3D Shapes



Task 5 - Description

Shape	Description
Sorted by Edges and V	ertices
Sphere and "egg like"	- Shapes with no edges and no vertices (corners).
shapes	- Shapes with edges but no vertices (e.g., flying saucer).
	- Shapes with vertices but no edges (e.g., a football).
Sorted by Faces and Su	irfaces
Polyhedron	 Shapes made of all faces (a face is a flat surface of a solid). If all surfaces are faces, all the edges will be straight lines). Some combination of faces and rounded surfaces (cylinders are examples but this is not a definition of a cylinder). Shapes with curved surfaces. Shapes with and without edges and with and without vertices. Faces can be parallel. Parallel faces lie in places that never intersect.
Cylinders	
Cylinder	Two congruent, parallel faces called <i>bases</i> . Lines joining corresponding points on the two bases are always parallel. These parallel lines are called elements of the cylinder.
Right cylinder	A cylinder with elements perpendicular to the bases. A cylinder that is not a right cylinder is an oblique cylinder.
Prism	A cylinder with polygons for bases. All prisms are special cases of cylinders.
Rectangular prism	A cylinder with rectangles for bases.
Cube	A square prism with square sides.
Cones	
Cone	A solid with exactly one face and a vertex that is not on the face. Straight lines (elements) can be drawn from any point on the edge of the base to the vertex. The base may be any shape at all. The vertex need not be directly over the base.
Circular cone	Cone with a circular base.
Pyramid	Cone with a polygon for a base. All faces joining the vertix are triangles. Pyramids are named by the shape of the base: <i>triangular</i> pyramid, <i>square</i> pyramid, <i>octagonal</i> pyramid, and so on. All pyramids are special cases of cones.

Task 5 (independent)

Are all the three-sided shapes on this piece of ngatu triangles?



Why or why not? Be ready to explain and justify your answer using all the three-sided figures on this piece of ngatu.

Now that you are becoming an expert in geometry I think that you are ready to develop a mathematical argument.

• You have to decide whether the following statements could be described as:

Always true. Sometimes true. Never true.

- 1. A hexagon has six equal length sides.
- 2. Triangles have a line of symmetry
- 3. Squares have two diagonals that meet at right angles
- 4. Cutting a corner off a square makes a pentagon
- 5. The base of a pyramid is a square
- 6. When you cut off a piece from a 2D shape, you reduce the area and perimeter
- 7. The number of lines of symmetry in a regular polygon is equal to the number of sides
- 8. Quadrilaterals can be cut into two equal triangles

Make sure that you can justify your reasoning for each decision you make.

Now, for the sometimes statements can you develop an explanation of when the statements are true...or rewrite them so that they are always true... or never true.

Task 6 (independent)

Regular polyhedrons are shapes that have all sides equal in length and all inside angles are equal.

Irregular polyhedrons are 2-D shapes that have straight sides that are not equal to each other and angles that are not equal to each other.

Fill in the missing details. Draw examples of what the following might look like. (see next page)

Task 6: Independent (continued)

number of sides & angles	name	Draw an example of regular polygon	example of irregular polygon
3	Triangle		
3	Triangle		
	Kite		
4	Quadrilateral		
5	Pentagon		
	Hexagon		
	Square		
7			
	Octagon		
	Nonagon		
10			

When the box makers were designing these cuboids, they drew a 2D representation of their net. What 2D shapes did they draw?

Look carefully at one of the cuboids and imagine what it would look like flattened out as a net. Talk about how many faces it will have and how many will be congruent.

Draw what you think it will look like as a net. Remember that when you fold the net up it needs to make a 3D cuboid and so you need to draw all the faces.

Task 7 (independent)

Type of triangle	Drawing	Properties
Obtuse		
Isosceles		
Scalene		
Right Angle		
Equilateral		
Equiangular		
Acute		

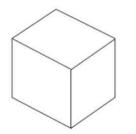
Draw and describe the properties of the following triangles:

James is in a soccer shop with his mother and while he waits for her he starts looking closely at the soccer balls and imagining what their net might look like.



Have a close look at these soccer balls. What do you notice about the 2D shapes which would be used in the net of a soccer ball? Draw a miniature net and test it out to see if your net makes a miniature soccer ball.

Task 8 (independent)



Draw all the possibilities for nets for a cube.

Test to see whether all of your nets make cubes.

You are a constructor.

- 1. Can you draw a net for a pyramid?
- 2. Can you draw a net to make a triangular packet to hold six tennis balls?
- 3. Can you draw a net for this chocolate box?



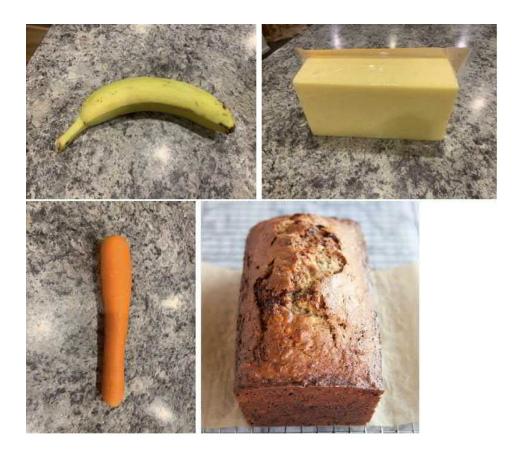
4. Can you draw a net for this Toblerone bar?



Draw a net for another shape of your choice.

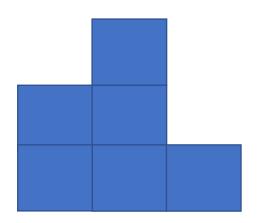
Task 9 (independent)

If you cut these objects in half, what would the slice look like for each?



Constructors ready for another challenge?

Here is a shape made with linking cubes. When you look at it from one side, it looks like this.



What do you think the whole structure looks like?

Before you explore and experiment with your cubes can you visualise what you think it looks like. Make a drawing of what you think it might look like on isometric paper and then build it with the cubes.

Now look at your construction from a different view. Draw this view on isometric paper.

How many different views of the construction can you draw on isometric paper?

Task 10 Connect Resource



Task 10 (independent)

Look at this siapo. Make a table and describe all the attributes of the different geometric shapes you see.



Task 11 (Optional)

Ready for a new challenge?

Shuffle the cards and place four of them horizontally and four of them vertically on the grid.

Your challenge is to draw a quadrilateral in each empty square, so that the quadrilateral has both the properties at the top of the column and at the start of the row.

There might be some that may not be possible!

Use squared paper and isometric paper to help you find areas and angles.

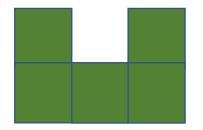
Can you make a 5 by 5 grid that fit the cards and that the students can draw the quadrilateral in the empty space?

Task 11 – Resource

Has more than two equal sides	Has just one right angle	Has just one obtuse angle	Has no equal angles	Has no equal sides
Has all angles equal	Has rotational symmetry	Has two pairs of parallel sides	Has just one axis of symmetry	Has area of 2 units
Has a reflex angle	Has just one pair of parallel sides	Has just two equal sides	Has just one pair of opposite angles equal	Has more than one right angle
Has more than two equal angles	Has just two equal angles	Has more than one axis of symmetry	Has area of 1 unit	Has just two right angles

Task 11 (Optional Independent Task)

Here is a shape made with linking cubes. When you look at it from one side, it looks like this.



When you look at it from one side, it looks like this.

What do you think the whole structure looks like?

Before you explore and experiment with your cubes can you visualise what you think it looks like. Make a drawing of what you think it might look like on isometric paper and then build it with the cubes.

Now look at your construction from a different view. Draw this view on isometric paper.

How many different views of the construction can you draw on isometric paper?

Task 12 (Optional)

Final constructor challenge: Draw these different polygons and keep noticing what changes and what stays the same.

- 1. This polygon has four sides.
- 2. This polygon has four right angles.
- 3. The length of this polygon is twice its width.
- 4. The area of the polygon is 18cm².
- 5. This polygon has two congruent sides.
- 6. The interior angles of this polygon have the sum of 180 degrees.
- 7. One side of this polygon is double the length of another side.
- 8. This polygon has a perimeter of 40cm.
- 9. The shortest side is half the length of the longest side.
- 10. The two longest sides of this polygon meet in a 30 degree angle.

Dotty Paper

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Isometric Dotty Paper

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