DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

Number and Algebra: Patterns and Relationships

Level 3 (Year 5/6)

Teacher Booklet

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 1	Litea has a giant bag of M & Ms. She likes to eat her M & Ms in a specific order: red, orange, green, yellow, blue, brown. What will be the colour of the 87th M & M that she eats? Find two different ways of solving the task and representations to prove your solutions.
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a sequential pattern. • Explain and justify the pattern in relation to ordinal aspects of number. • Identify the element for a sequential pattern for far terms. • Explain that a pattern has consistency. • Develop generalisations expressed in words and symbols related to a sequential pattern.
Mathematical language	Sequence, element, rule, unit of repeat, position.
Sharing back/Connect	Select students to share who use a grouping strategy to solve the task. For example, students might notice that a blue M & M is always a multiple of 6 and use this to get close to 87 by stating that 60 or 90 would be blue and then using the pattern structure. **Connect:**

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	What do you notice about all of the brown M & Ms in relation to their pattern position? What rule could you use to find the location of every brown M & M? What rule could you use to find the location of every orange M & M?
Teacher Notes	• During the launch, ask students to share where they see patterns in their lives. Reinforce discussion that patterns have sequential elements. • Have multilink cubes available for the students to model the pattern. • Facilitate the students to model the pattern in a way that highlights the structure of the pattern:  Use the model to support students to notice that certain elements align with specific numbers or multiples. • Use the term unit of repeat for the core of the pattern. • Notice students who use grouping or identify multiples to solve the task. • Expect students to represent using blocks and numbers and to identify the number patterns associated with the specific colours (similar to choral counting). • For the connect, students might notice that the blue M & Ms are all multiples of 6 so the rule for blue M & Ms would be multiply by 6. Ask the students for the rule in words and then model writing the rule using variables or informal variables (e.g., $6 \times a$ or $6 \times \bigstar$ or introduce shorthand of $6a$). For the orange M & M, you could multiply by 6 and add 2 or multiply by 6 and subtract 4 (e.g., $6a + 2$ or $6b - 2$).
Independent Tasks	At Te Oro the Siva Samoa group is learning a maulu'ulu. As part of the dance, they used these movements: tap, tap, arm, arm, arm, arm, arm, clap They repeat these moves lots of times throughout the dance.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

<table>
<tr><td></td><td>If they repeat the moves two times, how many taps would there be? How many arms would there be? How many claps would there be?

Complete the table below:

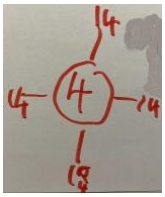
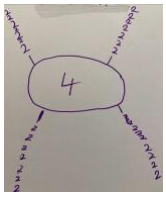
<table>
<tr><td>Number of movement sequence</td><td>Tap</td><td>Arm</td><td>Clap</td></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>2</td></tr>
<tr><td></td><td></td><td>20</td><td></td></tr>
<tr><td>9</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>15</td></tr>
<tr><td></td><td>40</td><td></td><td></td></tr>
<tr><td>26</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>30</td></tr>
</table>

What rules could you use to find the number of specific movements for any number of movement sequences?</td></tr>
<tr><td>Anticipations</td><td></td></tr>
</table>

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 2	**Tivaevae**  ***A group of Mamas are working on a tivaevae design.***  They want to turn the pattern from the cushion cover into a bedspread and keep the pattern the same. How do you see the pattern growing? Represent the pattern using numbers. How many leaves would the next position have? How many leaves would the 7th position have? How many leaves would the 12th position have? Represent this using a diagram and numbers.
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.

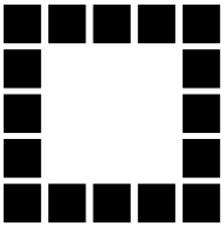
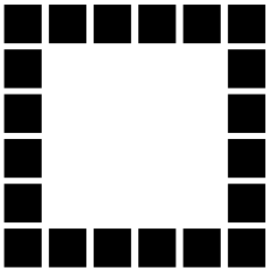
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Identify the growing element and constant in a linear growing pattern. • Represent the growing element and the constant using numbers and symbols. • Represent a growing pattern in a table of data. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, table of data.
Sharing back/Connect	Select students to share who have represented the pattern using colours or number and generalised this growing in different ways. The images below represent the ways that Position 7 may be represented.  A  B  C  D **Connect:** Describe how you would find the leaves for the 51st position. What is a rule that you could use to find the number of leaves no matter the position for each of the ways that the pattern is represented?
Teacher Notes	• During the launch, ask students to share different types of patterns that they know from their culture or every-day life. • Facilitate the students to notice the constant of four leaves in the middle and that the leaves around the edge change as the pattern grows. • Notice students who use recursive generalisation (add eight each time) in relation to how the pattern grows and support them to move towards an explicit generalisation (what is the same as adding eight multiple times?). • For the connect, the following generalisations relate to each way of seeing the patterns. A) & B) $4(7 \times 2) + 4$ or $4(n \times 2) + 4$; C) & D) $8b + 4$. Introduce letters as variables which can represent any number. • Expect students to represent using numbers and representations. Students could also be introduced to a table of data to show a structured way to see the relationship between the position and the total number of leaves.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	• For the independent task, have materials available if needed.
Independent Tasks	Litea has a giant bag of M & Ms. She likes to eat her M & Ms in a specific order: green, red, brown, blue, yellow, orange. What will be the colour of the 55^{th} M & M that she eats? Find two different ways of solving the task and representations to prove your solutions. What do you notice about all of the green M & Ms in relation to their pattern position? What rule could you use to find the location of every green M & M? What rule could you use to find the location of every yellow M & M?
Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 3	Melvin is designing a square garden plot with a tile border. He is wondering how many tiles he will need for gardens of different sizes.  Garden 3  Garden 4 Draw what the square garden plot would look like for Garden 1 and Garden 2. How many tiles would be used for Garden 5? How many tiles would be used for Garden 8? What do you notice? Represent the parts of the pattern that are staying the same and the part of the pattern that changes as it grows using different colours. How many tiles would be used for Garden 15?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Identify the growing element and constant in a linear growing pattern. • Represent the growing element and the constant using numbers and symbols. • Represent a growing pattern in a table of data. • Develop generalisations expressed in words and symbols related to a growing pattern. • Test whether a generalisation works for a growing pattern.
Mathematical language	Position, element, rule, table of data.
Sharing back/Connect	Select students to share who have developed a generalisation including both recursive and explicit generalisations. If no students have developed an explicit generalisation, then model how to turn the recursive generalisation into a rule. **Connect:** Melvin thinks that he has found some different ways to work out the number of tiles for every garden. Check his ideas and see whether the rules work or not: 1) Garden number + garden number + garden number + garden number + four ($d + d + d + d + 4$) 2) Garden number + garden number multiplied by 4 ($a + a$) × 4 3) Garden number multiplied by 4 + 4 ($4j + 4$)
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Have squares available for students to construct the pattern. • Facilitate the students to notice that the pattern has a constant (the part that stays the same) and a part that grows. Support them to colour the part that stays the same in one colour and the part that grows in a different colour and link their number sentences to this. • Notice students who use a table of data or a structured way of tracking the number of squares as the pattern grows. Model how to use a table of data if needed.

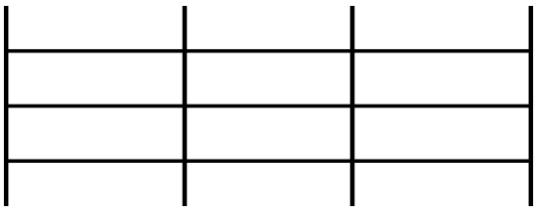
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

<table>
<tr><td></td><td>

- Expect students to move from recursive generalisation (it increases by + 4 after the first pattern) to an explicit generalisation.
- For the independent task, have ice-block sticks available.

</td></tr>
<tr><td>Independent Tasks</td><td>

Mele is helping her Mum build a fence around their house.



This is the third section of the fence.

Build and then draw what the first, second, and fourth section would look like.

Complete the table:

<table>
<tr><th>Fence section</th><th>Number of posts</th></tr>
<tr><td>1</td><td></td></tr>
<tr><td>2</td><td></td></tr>
<tr><td>3</td><td></td></tr>
<tr><td>4</td><td></td></tr>
<tr><td>5</td><td></td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>10</td><td></td></tr>
<tr><td>13</td><td></td></tr>
<tr><td>21</td><td></td></tr>
<tr><td>25</td><td></td></tr>
</table>

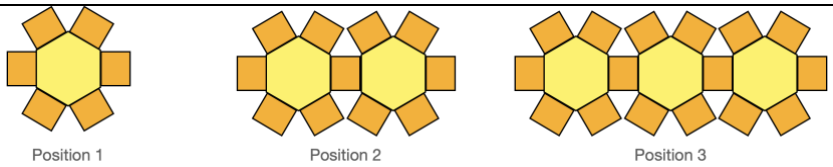
What patterns do you notice?

Can you develop a rule for the number of posts for the fence section of any size?

</td></tr>
<tr><td>Anticipations</td><td></td></tr>
</table>

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

<table>
<tr>
<td>Task 4</td>
<td>


Use the shape cards to build Position 4 and 5.

Complete the table:

<table>
<tr><th>Position Number</th><th>Hexagons</th><th>Squares</th><th>Total pieces</th></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td></tr>
<tr><td>7</td><td></td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td><td></td></tr>
</table>

Identify three patterns across the table and three patterns down the table.

How many hexagons would there be for the 10th sequence?
How many squares would there be for the 10th sequence?
How many shapes in total would there be for the 10th sequence?
</td>
</tr>
<tr>
<td>Big ideas</td>
<td>Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule.
Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm).
A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows).
In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern’s rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed.
Generalisations can be expressed with both words and symbols.
Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.</td>
</tr>
<tr>
<td>Curriculum links</td>
<td>NA-2-8: Find rules for the next member in a sequential pattern.
NA-3-8: Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find</td>
</tr>
</table>

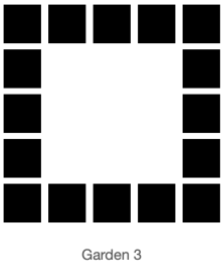
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Represent a growing pattern in a table of data. • Identify patterns in a table of data that draw on single variational thinking or co-variational thinking. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, unit of repeat, variable.
Sharing back/Connect	Select students to share who develop explicit generalisations using multiplication and relationships to work out the different number of shapes who identify co-variational patterns in the table. **Connect:** What rules can you use to find the different types of shapes? Hexagons: Squares: Total number of shapes:
Teacher Notes	• Have shape blocks and counters available. • If students have difficulty making the pattern, support them to notice by putting their pattern next to a picture of the correct pattern and ask them to identify what is the same and what is different. • Facilitate the students to notice patterns in the table vertically may include sequential or single variational thinking (e.g., the hexagons increase by one, the squares increase by 5 each time) or horizontally co-variational or relational thinking (e.g., the total number of shapes is the squares plus the hexagon or the number of squares is the number of hexagons × 2 + 1). • If students use recursive patterns (+ 5 for the squares) facilitate them to connect this to multiplication. Facilitate the students to notice that the square at the beginning is the constant and stays the same as the pattern grows. This can be linked to the + 1 in the rule for the total number of squares. • Have square shapes available for the independent activity.

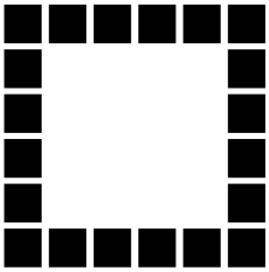
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Independent Tasks

Melvin is designing a square garden plot with a tile border. He is wondering how many tiles he will need for gardens of different sizes.



Garden 3



Garden 4

Complete the table:

Garden	**Number of tiles**
1	
2	
3	
4	
5	
8	
10	
11	
16	
21	

Melvin has 108 tiles, what garden number can he make? Would he have any left over?

Anticipations

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 5	Pacific people are voyagers and developed clever ways construct vaka. An example is the vaka at Matauala Hall in Porirua which has a pattern where the fauato (coconut fibre twine) joins the planks. As the vaka gets longer, it needs more fauato.  Tahi  Lua Can you build and draw the next pattern? How many pieces of fauato would there be if the pattern went up to iva (nine)? How many pieces of fauato would there be if the pattern went up to hefuluiva (nineteen)? What about ivahefulu (ninety)?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	NA-3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Represent a growing pattern in a table of data. • Represent a growing pattern on a graph. • Identify patterns in a table of data that draw on single variational thinking or co-variational thinking. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, unit of repeat, variable, graph.
Sharing back/Connect	Select students to share who recognise that the pattern increases by 6 each time and have used multiple representations (e.g., diagrams, tables of data, numbers) to show their reasoning. **Connect:** Represent how the pattern grows using a line graph on the graph (squared) paper. What a rule you could use to find the number of pieces of fauato for any number of the pattern?
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Have counters and ice-block sticks to construct the pattern and grid paper to draw the graphs for the connect. • Expect students to represent using diagrams, a table of data, numbers, and equations. If necessary for the connect, model how to create a line graph to represent how the pattern grows. • For the independent activity, have graph (squared) paper available.
Independent Tasks	Tane is cutting up pieces of string for weaving. If he cuts the string in half, he has two pieces. If he cuts the two pieces together again, he has four pieces. If he keeps putting the pieces together again before cutting how many pieces would he get with….

	Three cuts? Four cuts? Use a table of data and a graph to show how many pieces of string Tane would have if he cut it up to 12 times. What rule could Tane use to work out how many pieces he would get if no matter how many times he cut the string?
Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

<table>
<tr><td>Task 6</td><td>Amazon Prime is currently offering two deals for watching movies.

Plan A costs $18 monthly membership plus $2 per movie.
Plan B costs $9 monthly membership plus $3 per movie.

Use a number sentence to represent the two deals.

Show the results for Plan A and Plan B in a table.

<table>
<tr><td>Number of movies</td><td>Deal 1</td><td>Deal 2</td></tr>
<tr><td>1</td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td></tr>
<tr><td>7</td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td></tr>
<tr><td>9</td><td></td><td></td></tr>
<tr><td>10</td><td></td><td></td></tr>
</table>

Which deal is better?

What advice would you give to someone considering both the plans?</td></tr>
<tr><td>Big ideas</td><td>Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule.
Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm).
A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows).
In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern’s rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed.
Generalisations can be expressed with both words and symbols.
Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.</td></tr>
<tr><td>Curriculum links</td><td>NA-2-8: Find rules for the next member in a sequential pattern.</td></tr>
</table>

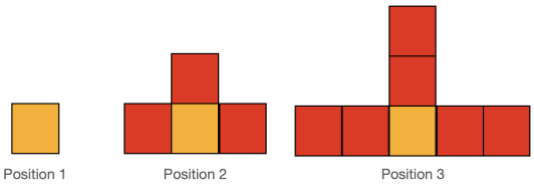
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	NA-3-6: Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Represent a situation with unknowns using a number sentence. • Solve number sentences with different quantities. • Represent a function in a table of data. • Compare functional situations for different quantities.
Mathematical language	Unknown, variable, table of data.
Sharing back/Connect	Select students to share who can represent the unknown using a letter or informal variable (shape) and use the table of data to compare the results. **Connect:** Ask students to show the results for both deals on a line graph using two different colours. Ask them to discuss how the graph could help them make predictions.
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Facilitate the students to notice that the different deals with vary depending on the constant (monthly price) and individual movie price. . • Expect students to represent using number sentences and a variable and a table of data. • For the independent task, have shapes, ice-block sticks and grid paper for the students to work with.
Independent Tasks	Develop a growing pattern to match these rules: Squares = $(2 \times n) + 1$ Ice-block sticks = $(5 \times h) + 3$ Tiles = $(3 \times g) - 2$ Develop your own growing patterns and write a rule to match them.

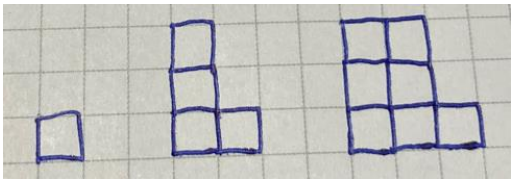
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

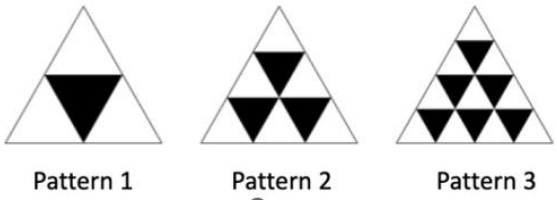
Task 7	Tayla builds a pattern with the blocks:  Use the square shapes to create Position 4, Position 5, and Position 6 and draw these. How many blocks would they each use? Tayla is wondering what position she could build if she had 85 blocks and whether she would have any blocks left over?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Represent a pattern using objects, drawings or a diagram. • Identify patterns related to growing patterns. • Use recursive methods to predict the next member of a sequence in a linear pattern. • Develop generalisations expressed in words and symbols related to a linear pattern.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

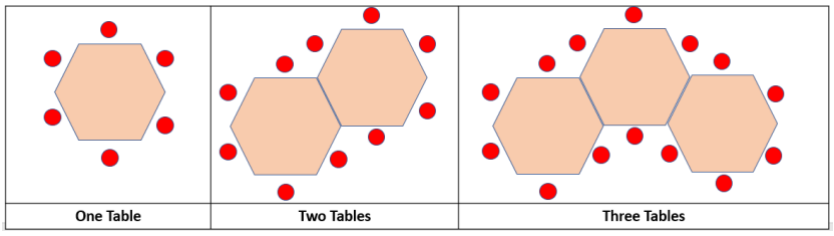
Mathematical language	Position, element, rule.
Sharing back/Connect	Select students to share who develop generalisations (recursive or explicit) that identify that the pattern grows by 3 each time. If no students identify that the generalisation is related to × 3 then model this for them. **Connect:** Provide students with blocks or squares and ask them to use the same number of blocks but change the structure to clearly show the ×3 relationship. For example;  Ask the students to use this to develop a rule.
Teacher Notes	• Have square shapes or blocks. • If students have difficulty creating or continuing the pattern ask them to re-create the earlier positions and compare them to see what is the same and different. • Notice students who use a table of data or a structured way of tracking the number of squares and total number of shapes as the pattern grows. • For the independent task, provide students with graph (squared) paper.
Independent Tasks	Ta'ase and Lina are selling different types of chocolate bars to fundraise for camp. Ta'ase gets $3 for each chocolate bar. Lina has saved $15. Additionally, for each chocolate bar she sells, she gets $2. Write a number sentence to represent each situation. Use a table of data and graph to show when Ta'ase and Lina will have the same amount of money and how many chocolate bars Ta'ase will need to sell to have more money.
Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 8	Mātua and Whaea are looking at designs for the Kapa Haka uniform. They like the triangular patterns which represent peaks and valleys.  What do you notice about the pattern and how it grows? Draw or describe Pattern 4 and Pattern 5. Can you show the way the pattern grows for the different elements? Think about the white triangles, the black triangles, and the total number of triangles. Represent your ideas using pictures, numbers, and a table of data. What rules could Mātua and Whaea use to find the different elements?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	NA-4-6: Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Identify the growing element in a growing pattern. • Represent the growing element using numbers and symbols. • Develop generalisations expressed in words related to a growing pattern.
Mathematical language	Element, rule, table of data
Sharing back/Connect	Select students to share who have noticed different patterns in relation to the different elements of the patterns. **Connect:** Describe what Pattern 100 would look like? What would be a quick way to work out how many black triangles in total you would need for Pattern 100? What would be a quick way to work out how many white triangles in total you would need for Pattern 100? What would be a quick way to work out how many triangles in total you would need for Pattern 100?
Teacher Notes	• During the launch, ask students to share different types of patterns that they know from their culture or every-day life. • Facilitate students to notice that the pattern is made up of different elements including black triangles, white triangles, and the total number of triangles. • Position the students to use different representations and diagrams including numbers and a table of data. A table of data with three columns may be introduced or modelled to support the students. • For the independent activity, have hexagons and counters available for the students to model the pattern.
Independent Tasks	Eva and Hone have been put in charge of organising tables and seats for their school graduation night dinner. They've worked out that 242 people are coming. 

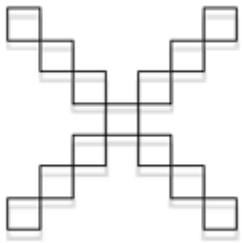
Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

	How many people can be seated at 6 tables, 12 tables, 24 tables? Represent how the pattern grows using a table of data or graph. How many tables do Eva and Hone need to organise for 242 people? Can you find the relationship between the number of tables and chairs in words or symbols and justify how the rule works with your representation.
Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 9 (optional task)	Tasha is making smiley face rods for market day by joining cubes together and putting smiley face stickers on each side that you can see:  Rod 1 Rod 2 Rod 3 How many smiley face stickers would Tasha need for rods of length 1 – 10? How many stickers would Tasha need for a rod of length 27? How many stickers would Tasha need for a rod of length 40? How many stickers would Tasha need for a rod of length 111? What rule could Tasha use to work out how many stickers she would need for a rod of any length?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, table of data
Sharing back/Connect	Select students to share who recognise that the pattern increases by 4 each time and have used multiple representations (e.g., physical cubes, diagrams, tables of data, numbers) to show their reasoning. **Connect:** Represent how the pattern grows using a line graph on the graph (squared) paper. Explain how your rule works using the graph.
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Have multi-link cubes to construct the pattern and grid paper to draw the graphs for the connect. • Expect students to represent using diagrams, a table of data, numbers, and equations. If necessary for the connect, model how to create a line graph to represent how the pattern grows.
Independent Tasks	 This is Pattern 3. Draw what you think Pattern 2 and Pattern 1 would look like. How many different patterns can you see in this drawing? Show all the patterns that you can see. Continue the pattern for Pattern 4 – 10. Use a table of data to represent the pattern and explain the patterns that you have found.
Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

Task 10 (optional task)	How many telephone calls could be made among 5 friends if each person spoke with each friend exactly once on the telephone? How many telephone calls would there be if there were 6 friends? Seven friends? Eight friends? Twenty friends? One hundred friends? Organize your data in a table. Describe any relationship you see between the number of phone calls and the number of friends in the group.
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Represent a pattern using drawings or a diagram. • Identify patterns related to quadratic functions. • Represent a quadratic pattern in a table of data. • Use recursive methods to predict the next member of a sequence in a non-linear pattern. • Develop generalisations expressed in words and symbols related to a quadratic pattern.
Mathematical language	Position, element, rule, variable, graph.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

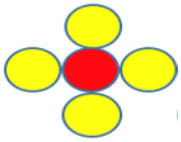
Sharing back/Connect	Select students to share who develop representations that clearly show all of the phone calls with different amounts of people. **Connect:** How can you develop a representation that proves that you have found all of the phone calls?
Teacher Notes	• During the launch, support students to understand that each friend can only speak to the other friend once. • Facilitate the students to develop a clear representation to ensure that they can correctly count the number of phone calls for different numbers of friends. • This task involves a quadratic function so it is important that the students have ample opportunity to explore the patterns.
Independent Tasks	Complete the following assessment tasks (attached at the end of the document) as the independent activity: A4: Flower pattern A4A: Square tiles pattern
Anticipations	

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

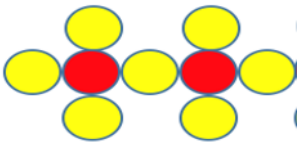
DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

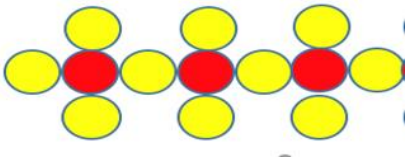
ALGEBRA: LEVEL 3 Task A4



Position One `



Position Two



Position Three

Look at this pattern. Can you:

- Show how the yellow circles grow using a picture, a table, ordered pairs, and/or a graph.
- Describe the pattern in words and/or numbers.
- Show how you work out the number of yellow circles for the following pattern numbers: Pattern 5; Pattern 10; Pattern 50.
- Write the rule for the number of yellow circles in words or symbols.

Level 3 Year 5/6: Number and Algebra: Patterns and Relationships

DMIC

DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

ALGEBRA: LEVEL 3 Task A4A

Position five

Look at this pattern. Can you:

- Draw position four, three, and six.
- Show how the grey squares grow using a table of data and/or a graph.
- Describe the pattern in words and/or numbers.
- Show how you work out the number of grey squares for the following pattern numbers: Pattern 5; Pattern 10; Pattern 50.
- Write the rule for the number of grey squares in words or symbols.