DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES

Number and Algebra: Patterns and Relationships

Level 4 (Year 7/8)

Teacher Booklet

Level 4 Year 7/8: Number and Algebra: Patterns and Relationships

Task 1	 How many different patterns can you see in this drawing? Use diagrams to show all the patterns that you can see. How would you draw the next position? How would you draw the 10th position? How would you draw the 25th position? How many stars would you need for the fourth position? How many stars would you need for the 10th position? How many stars would you need for the 25th position? Represent what you have found in a table of data.
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a linear growing pattern. • Identify the growing element and constant in a linear growing pattern. • Represent the growing element and the constant using numbers and symbols.

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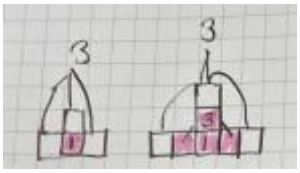
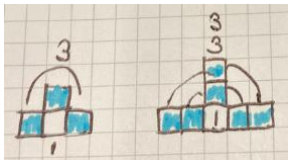
	• Represent a growing pattern in a table of data. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, table of data.
Sharing back/Connect	Select students to share who have represented the pattern using colours or numbers and generalised this growing in different ways. If students are using recursive reasoning then model how this could be changed to multiplication.  or  **Connect:** What is a rule that you could use to find the number of elements for any position? Link your rule to a representation.
Teacher Notes	• Have squares available for students to construct the pattern. • Facilitate the students to notice that the pattern has a constant (the part that stays the same) and a part that grows. Support them to colour the part that stays the same in one colour and the part that grows in a different colour. • Notice students who use a table of data or a structured way of tracking the number of squares as the pattern grows. Model how to use a table of data if needed. • Expect students to move from recursive generalisation (it increases by + 3 after the first pattern) to an explicit generalisation. • For the connect, introduce letters as variables which can represent any number. • For the independent task, have multi-link cubes available to model the task.
Independent Tasks	Tasa has a giant bag of M & Ms. He likes to eat his M & Ms in a specific order: orange, green, red, yellow, blue, brown. What will be the colour of the 93rd M & M that he eats? Find two different ways of solving the task and representations to prove your solutions. What do you notice about the brown M & Ms in relation to their pattern position?

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	What rule could you use to find the location of every brown M & M? What do you notice about the yellow M & Ms in relation to their pattern position? What rule could you use to find the location of every yellow M & M?
Anticipations	

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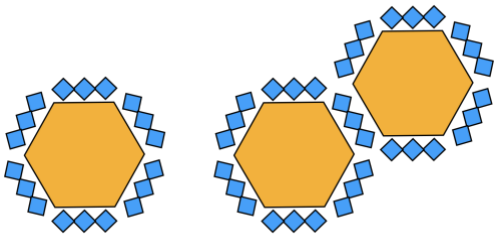
Task 2	 A group of Mamas meet at Manihiki Hall every week to sew tivaevae. Tivaevae ta'orei is a type of patchwork quilt made with small squares of fabric. This is a beautiful treasure which takes many years to make. Mama Jane is sewing a design which uses small diamonds around the border.  How many diamonds would Mama Jane need for pattern one to pattern five? How many diamonds would Mama Jane need for pattern ten? Represent the parts of the pattern that are staying the same and the part of the pattern that changes as it grows using different colours. How could Mama Jane work out how many diamonds she would need for pattern 100?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change,

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	representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Identify the growing element and constant in a linear growing pattern. • Represent the growing element and the constant using numbers and symbols. • Represent a growing pattern in a table of data. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, table of data.
Sharing back/Connect	Select students to share who have developed a generalisation to find pattern 100 including both recursive and explicit generalisations. If no students have developed an explicit generalisation, then model how to turn the recursive generalisation into an explicit generalisation. **Connect:** Mama Jane has 240 diamonds that are cut and ready to be sewed. She would like to know how many hexagon patterns she can create and whether she would have any diamonds left over.
Teacher Notes	• During the launch, ask students to share different types of patterns that they know from their culture or every-day life. • Notice students who use recursive generalisation (add 15 each time) in relation to how the pattern grows and support them to move towards an explicit generalisation (what is the same as adding 15 multiple times?). • Expect students to represent using numbers and representations. Students could also be introduced to a table of data to show a structured way to see the relationship between the position and the total number of leaves. • For the independent task, provide students with ice-block sticks to model the pattern.
Independent Tasks	Tasa is helping his Mum build a fence around their property. They are trying to work out how many posts they will need. The photo below shows the third section of the fence.

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Build and then draw what the first, second, and fourth section would look like.

Complete the table:

Fence section	Number of posts
1	
2	
3	
4	
5	
8	
10	
13	
21	
25	

What patterns do you notice?

Can you develop a rule for the number of posts for the fence section of any size?

Anticipations

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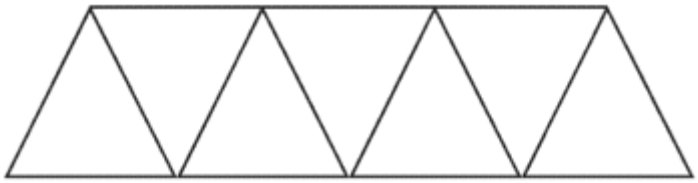
Task 3	Beams are used as a support for different types of bridges. The beams are constructed using steel rods. The number of rods used to construct the bottom of the beam determines the length of the beam. Below is a beam of length 4.  1 rod 2 rods 3 rods 4 rods Make and then draw the beams of length 2, 3, and 5. How many rods are needed to make each? How many rods would you need to make a beam of length 10? Of length 20? Of length 45?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing linear pattern in both directions.

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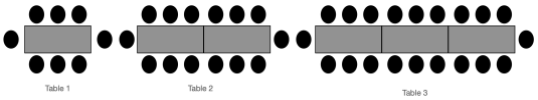
	• Represent a growing pattern in a table of data. • Represent a growing pattern on a graph. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, variable, graph.
Sharing back/Connect	Select students to share who recognise that the pattern increases by 4 each time and have used multiple representations (e.g., diagrams, tables of data, numbers) to show their reasoning. **Connect:** Represent how the pattern grows using a line graph on the graph (squared) paper. What rule could you use to find the number of rods for any number of the pattern?
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Have ice-block sticks to construct the pattern and grid paper to draw the graphs for the connect. • Expect students to represent using diagrams, a table of data, numbers, and equations. If necessary for the connect, model how to create a line graph to represent how the pattern grows. • For the independent activity, have graph (squared) paper available.
Independent Tasks	Emmy and Astyn are organising tables and seats for a family reunion dinner. They've worked out that 186 people are coming.  How many people can be seated at 5 tables, 11 tables, 22 tables? Represent how the pattern grows using a table of data or graph. How many tables do Emmy and Astyn need to organise for 186 people? Can you find the relationship between the number of tables and chairs in words or symbols and justify how the rule works with your representation.

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Anticipations	

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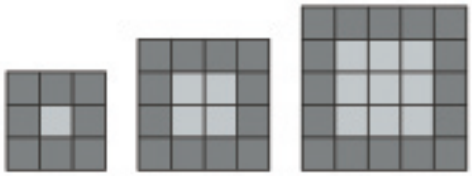
<table>
<tr><td>Task 4</td><td>Ayesha was helping to design a tile pattern and border for a square floor. She thought she could find a way to work out how many square tiles would be needed.

 
Pattern 1 Pattern 2 Pattern 3

What would the pattern look like for pattern 4 and 5?

Complete the table:

<table>
<tr><th>Pattern number</th><th>Border squares</th><th>Light grey squares</th><th>Total number of squares</th></tr>
<tr><td>1</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td><td></td></tr>
<tr><td>7</td><td></td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td><td></td></tr>
</table>

Identify three patterns across the table and three patterns down the table.</td></tr>
<tr><td>Big ideas</td><td>Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule.
Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm).
A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows).
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Generalisations can be expressed with both words and symbols.
Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.</td></tr>
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</table>

	relationships between successive elements of number and spatial patterns. **NA-4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a linear growing pattern. • Identify the growing element and constant in a linear growing pattern. • Represent the linear growing element and the constant using numbers and symbols. • Represent a growing pattern in a table of data. • Identify patterns in a table of data that draw on single variational thinking and co-variational thinking. • Develop generalisations expressed in words and symbols related to a growing pattern. • Test whether a generalisation works for a growing pattern.
Mathematical language	Position, element, rule, table of data.
Sharing back/Connect	Select students to share who identify co-variational patterns and relationships in the table. These can be used to develop explicit generalisations using multiplication to work out the different elements of the pattern. **Connect:** How could Ayesha turn the patterns that you noticed in the table into rules for different elements of the pattern?
Teacher Notes	• Have square shapes or squared paper to construct the pattern. • Facilitate the students to notice patterns in the table. Patterns vertically in the table may include sequential or single variational thinking (e.g., the border squares increase by 4 each time) or horizontally co-variational or relational thinking (e.g., the total number of shapes is the inside squares plus the border squares or the number of squares is the pattern number + 2 squared. • If students use recursive patterns (+ 4 for the squares) facilitate them to connect this to multiplication. Facilitate the students to notice that this pattern relates to square numbers. This pattern has a range of opportunities to develop different generalisations related to the same element of the pattern. For example, for the total amount of squares, you could either add the rule for the border and the rule for the inside squares or use $n + 2^2$.

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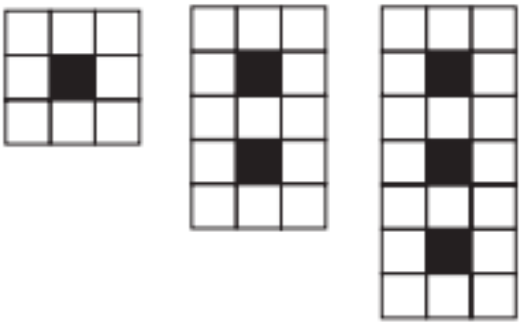
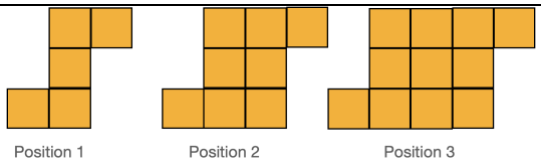
	• For the independent activity, have graph or squared paper available to model the pattern. Have square shapes available for the independent activity.
Independent Tasks	Sesimani was looking at design to make a mat. She would like your help to work out how many black squares and how many white squares she will need.  Position 1 Position 2 Position 3 How is the pattern growing? Use colours or number to show how you see it growing. Draw Position 4 and 5. How many white squares would there be for position 7? How many white squares would there be for position 14? How many white squares would there be for position 145? Can you work out a rule for the number of white squares? What would the rule be for the total number of squares?
Anticipations	

Task 5	 How is this pattern growing? Use numbers and colours to show how the pattern changes as it grows. How would you draw the next two positions? How many squares would the 10th position have? How many squares would the 12th position have? How many squares would the 100th position have?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
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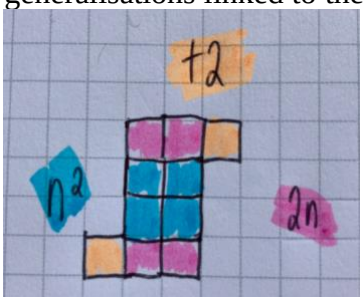
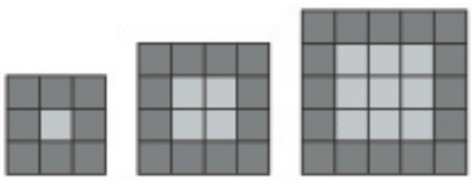
Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a linear growing pattern. • Identify the growing element and constant in a linear growing pattern.

	• Represent the linear growing element and the constant using numbers and symbols. • Represent a growing pattern in a table of data. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, table of data.
Sharing back/Connect	Select students to share who split or manipulate the pattern to identify and justify how different parts are growing as the pattern grows. This requires co-ordination of three different aspects. These can be used to develop explicit generalisations to work out the different elements of the pattern. **Connect:** Use squared paper or squares to construct the pattern and to justify how the pattern changes as it grows. Develop generalisations linked to the physical representation: 
Teacher Notes	• Have square shapes or squared paper to construct the pattern. • Facilitate the students to notice patterns in the physical representation of the pattern. For example, there is always a square in the middle of the pattern. • Expect students to develop a rich verbal description of how to make the pattern. • For the independent activity, have graph or squared paper available to model the pattern.
Independent Tasks	Ayesha was helping to design a tile pattern and border for a square floor. She thought she could find a way to work out how many square tiles would be needed.  Pattern 1 Pattern 2 Pattern 3

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	Can you find a rule to help Ayesha work out how many dark grey tiles she will need for the border for any pattern number? Can you find a rule to help Ayesha work out how many light grey tiles she will need for the middle for any pattern number? Can you find a rule to help Ayesha work out how many tiles she will need in total for any pattern number? Ayesha thinks that she has found some different ways to work out the number of tiles that would be needed for the border. Check her ideas and see whether the rules work or not: • $(d + d + d + d + 4) + d^2$ • $4g + g - g^2$ • $(n + 2)^2$
Anticipations	

<table>
<tr><td>Task 6</td><td>Destiny has joined a gym and is deciding on the best offer for her. The gym has two offers:

Offer A: $8 per class
Offer B: $50 for the first 5 classes of the month and then $4 for every additional class.

Develop a rule for each of the two offers.

Show the costs for Offer A and Offer B in a table.

<table>
<tr><td>Number of classes</td><td>Offer A</td><td>Offer B</td></tr>
<tr><td>1</td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td></tr>
<tr><td>7</td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td></tr>
<tr><td>9</td><td></td><td></td></tr>
<tr><td>10</td><td></td><td></td></tr>
</table>

Which offer is better?
What advice would you give to someone considering both the offers?</td></tr>
<tr><td>Big ideas</td><td>Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule.
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	relationships between successive elements of number and spatial patterns. **NA-4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Represent a situation with unknowns using a number sentence. • Solve number sentences with different quantities. • Represent a function in a table of data. • Compare functional situations for different quantities.
Mathematical language	Unknown, variable, table of data.
Sharing back/Connect	Select students to share who can represent the unknown using a letter to develop a rule and use the table of data to compare the results. **Connect:** Ask students to show the results for both offers on a line graph using two different colours. Ask them to discuss how the graph could help them make predictions.
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. (See the end of the Copy Masters Booklet) Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Facilitate the students to notice that the different offers will vary depending on the number of classes that Destiny takes each month. • Expect students to represent using number sentences and a variable and a table of data. • For the independent task, have square shapes and grid paper for the students to work with.
Independent Tasks	Melvin is designing a square garden plot with a tile border. He is wondering how many tiles he will need for gardens of different sizes.

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	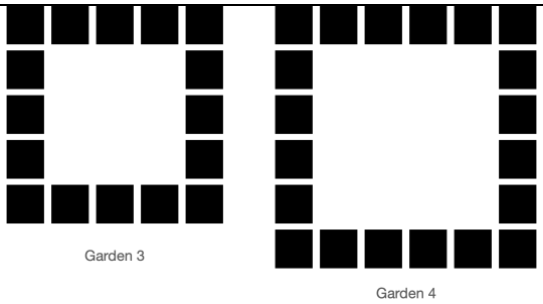 Draw what the square garden plot would look like for Garden 1 and Garden 4. How many tiles would be used for Garden 5? How many tiles would be used for Garden 8? What do you notice? Can you explain how you would find the number of tiles for Garden 100? If Melvin had 100 tiles, what garden number would he be able to make and would he have any tiles left over?
Anticipations	

Task 7	 Touch NZ are organising a community touch rugby event. Each team will play each other **once**. If there are four teams how many games of touch are there? If there are five teams how many games of touch are there? If there are six teams how many games of touch are there? Can you prove you know how many games there will be? Develop a clear representation to show all the games that will be played between each team.
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-2-8:** Find rules for the next member in a sequential pattern. **NA-3-6:** Record and interpret additive and simple multiplicative strategies, using words, diagrams, and symbols, with an understanding of equality. **NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.

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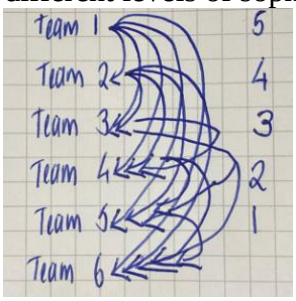
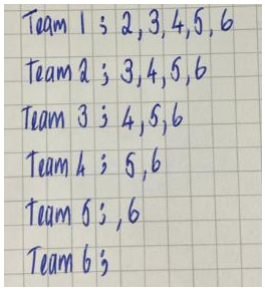
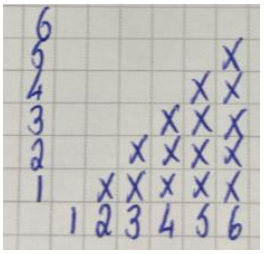
Learning Outcomes: Students will be able to:	• Represent a pattern using drawings or a diagram. • Identify patterns related to quadratic functions. • Represent a quadratic pattern in a table of data. • Use recursive methods to predict the next member of a sequence in a non-linear pattern. • Develop generalisations expressed in words and symbols related to a quadratic pattern.
Mathematical language	Position, element, rule, unit of repeat, variable, graph.
Sharing back/Connect	Select students to share who develop representations that clearly show all of the games that will be played for six teams with different levels of sophistication..  (A)  (B) **Connect:** Give the students grid paper and ask them to develop a representation for 4 teams, 5 teams, and 6 teams, using the format below:  Ask the students to discuss what they notice.
Teacher Notes	• During the launch, support students to notice that the teams do not play against themselves and they only play against another team once. • Have graph (squared) paper available for the connect. • Facilitate the students to develop a clear representation to ensure that they can correctly count the number of games for each number of teams. • This task involves a quadratic function so it is important that the students have ample opportunity to explore the patterns. The representation introduced in the connect will support students to consider a rule in the next lesson.

Level 4 Year 7/8: Number and Algebra: Patterns and Relationships

<table>
<tr><td></td><td>

- For the independent task, provide students with graph (squared) paper.

</td></tr>
<tr><td>Independent Tasks</td><td>

Smart data is currently offering two deals for phone data.

Deal 1 costs $14 per month for 1 GB plus $4 per GB.
Deal 2 costs $18 per month for 3 GB plus $5 per GB.

Represent each deal using a rule.

Show the results for Deal 1 and Deal 2 in a table.

<table>
<tr><td>Number of GB</td><td>Deal 1</td><td>Deal 2</td></tr>
<tr><td>1</td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td></tr>
<tr><td>7</td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td></tr>
<tr><td>9</td><td></td><td></td></tr>
<tr><td>10</td><td></td><td></td></tr>
</table>

Which deal is better?

What advice would you give to someone considering both the deals?

</td></tr>
<tr><td>Anticipations</td><td></td></tr>
</table>

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Task 8	 Touch NZ are organising a community touch rugby event. Each team will play each other once. If there are ten teams how many games of touch are there? If there are twenty teams how many games of touch are there? How would you find out the number of games for 100 teams? See if you can explain how to find all the games no matter how many teams there are. Make sure you can explain and justify why your rule works.
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Represent a pattern using drawings or a diagram. • Identify patterns related to quadratic functions. • Represent a quadratic pattern in a table of data. • Use recursive methods to predict the next member of a sequence in a non-linear pattern.

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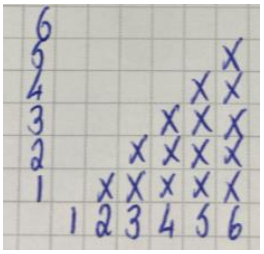
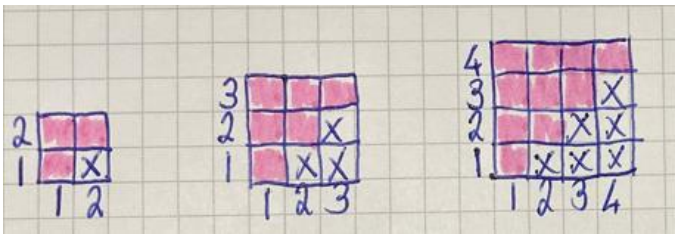
	• Develop generalisations expressed in words and symbols related to a quadratic pattern.
Mathematical language	Position, element, rule, unit of repeat, variable, graph.
Sharing back/Connect	Select students to share who use the representation from the connect to represent the number of games for ten teams:  (Example for six teams) Ask students to share if they have developed a rule or partial rule for the pattern. **Connect:** If students have developed a rule, then use the connect to ask the students to connect the rule to the representation above. Alternatively if no groups have developed the rule, show the following representation to the students and ask them to develop a rule from this:  $\frac{n^2(n-1)}{2}$
Teacher Notes	• During the launch, introduce the model for recording the number of games for the teams that was introduced in the connect in the previous lesson. • Have graph (squared) paper available. • Facilitate the students to develop a clear representation to ensure that they can correctly count the number of games for each number of teams. • This task involves a quadratic function so it is important that the students have ample opportunity to explore the patterns. For the independent task, have graph paper available.

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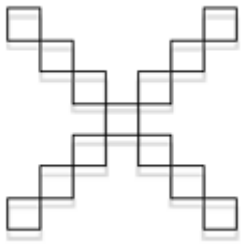
Independent Tasks	Tiana and Lyonel are selling different types of calendars to fundraise for the AIMs tournament. . Tiana has saved $16. Additionally for each calendar she sells, she gets $3. Lyonel gets $5 per calendar he sells. Write a rule to represent each situation. Use a table of data and graph to show when Tiana and Lyonel will have the same amount of money and how many calendars, Lyonel will need to sell to have more money.
Anticipations	

Task 9 (optional task)	Sima has saved some money (he only has dollars and no cents). His Grandma wants to reward him for some jobs that he has finished well. She offers him two options. Option 1: She will double his money Option 2: She will triple his money and then take away $7 Develop a rule for each of the two options. Which option is better? Justify your thinking using a representation (graph, table of data) What advice would you give to Sima?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.
Learning Outcomes: Students will be able to:	• Represent a situation with unknowns using a number sentence. • Solve number sentences with different quantities. • Represent a function in a table of data. • Compare functional situations for different quantities.
Mathematical language	Unknown, variable, table of data.

Level 4 Year 7/8: Number and Algebra: Patterns and Relationships

Sharing back/Connect	Select students to share who can represent the unknown using a letter to develop a rule and use the table of data or graph to justify their ideas. **Connect:** Represent this situation: Mele (Sima's sister) has $8 in her wallet and the rest of her money is in her bank account. Sima has exactly 3 times as much money as Mele has in her wallet. What can you say about the amounts of money that Mele and Sima have?
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Facilitate the students to notice that the different offers will vary depending on the amount of money that Sima has to begin with. This task also will provide students with experience in working with negative numbers. • Expect students to represent using number sentences and a variable and a table of data. • For the independent task, have square shapes and grid paper for the students to work with.
Independent Tasks	 This is Pattern 3. Draw what you think Pattern 2 and Pattern 1 would look like. How many different patterns can you see in this drawing? Show all the patterns that you can see. Continue the pattern for Pattern 4 – 10. Use a table of data to represent the pattern and explain the patterns that you have found.
Anticipations	

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Task 10 (optional task)	Tasha is making smiley face rods for market day by joining cubes together and putting smiley face stickers on each side that you can see:  Rod 1 Rod 2 Rod 3 How many smiley face stickers would Tasha need for rods of length 1 – 10? How many stickers would Tasha need for a rod of length 27? How many stickers would Tasha need for a rod of length 40? How many stickers would Tasha need for a rod of length 111? What rule could Tasha use to work out how many stickers she would need for a rod of any length?
Big ideas	Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics. The same pattern structure can be found in many different forms (e.g., numbers, shapes, colours, and rhythm). A pattern can be described using a rule or you can create a pattern from a rule. To find the rule for a pattern, you need to identify the unit of the pattern (what is repeated or what grows). In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements. Identifying the rule of a pattern brings predictability and allows generalisations to be developed. Generalisations can be expressed with both words and symbols. Variables are symbols that take the place of numbers, or ranges of numbers. They have different meanings depending on whether they are being used as representations of quantities that vary or change, representations of specific unknown variables, or placeholders in a generalised expression or formula.
Curriculum links	**NA-3-8:** Connect members of sequential patterns with their ordinal position and use tables, graphs, and diagrams to find relationships between successive elements of number and spatial patterns. **NA-4-1:** Use a range of multiplicative strategies when operating on whole numbers. **NA-4-6:** Use graphs, tables, and rules to describe linear relationships found in number and spatial patterns.

Level 4 Year 7/8: Number and Algebra: Patterns and Relationships

Learning Outcomes: Students will be able to:	• Reproduce a pattern using objects, drawings, or symbols. • Continue a growing pattern. • Develop generalisations expressed in words and symbols related to a growing pattern.
Mathematical language	Position, element, rule, variable, graph.
Sharing back/Connect	Select students to share who recognise that the pattern increases by 4 each time and have used multiple representations (e.g., physical cubes, diagrams, tables of data, numbers) to show their reasoning. **Connect:** Represent how the pattern grows using a line graph on the graph (squared) paper. Explain how your rule works using the graph.
Teacher Notes	• Before you launch the task, do a pattern quick image warm-up. Show the students the pattern for three seconds and ask them to describe it. Show the pattern again for three seconds and ask them to draw it. Display the pattern and ask them to draw the next term. • Have multi-link cubes to construct the pattern and grid paper to draw the graphs for the connect. • Expect students to represent using diagrams, a table of data, numbers, and equations. If necessary for the connect, model how to create a line graph to represent how the pattern grows.
Independent Tasks	Complete the following assessment tasks (attached at the end of the document) as the independent activity: A5: Triangles pattern A6: Necklace pattern
Anticipations	

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DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

ALGEBRA: LEVEL 4 Task A5

Pattern 1 Pattern 2 Pattern 3

Look at this growing pattern. Can you:

- Find the number of black triangles for the following pattern numbers: Pattern 6; Pattern 12; Pattern 57.
- Represent how the pattern grows using any or all of these: table, ordered pairs, graph.
- Write the rule for the number of triangles in words or symbols and justify how the rule works with your representation.
- Find the pattern number if the number of black triangles is 210.

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DEVELOPING MATHEMATICAL INQUIRY COMMUNITIES
ASSESSMENT TASK

ALGEBRA: Level 4-5 Task A6

Necklace patterns

First pattern **Second pattern** **Third pattern**

- Find the number of circular beads for the following pattern numbers: Pattern 6; Pattern 13; Pattern 57.
- Represent how the pattern grows using a table of data and graph.
- Write a rule for the number of circular beads in words or symbols and justify how the rule works with your representation.
- Write a rule for the total number of shapes in words or symbols and justify how the rule works with your representation.
- Find the pattern number if the number of circular beads is 248.