



# Conceptual Starters Phase TWO



# How To Guide

These conceptual starters have been designed and planned to meet learning intentions of the New Zealand curriculum. Whilst it is a large collection of starters there are many more starters that can be used in your mathematics programs.

## Each starter is:

- Designed to be used more than once.
- Written with a small number of other examples, however almost all starters could be adapted and used with a variety more different numbers, patterns and materials.
- Encouraging the use of mathematical practices.
- Supporting the use of dialogue and communication during these starters.
- Designed to be chosen intentionally and used to revisit or build upon concepts taught throughout the year.

## Mathematical Practices are:

- Making an explanation
- Making a justification
- Arguing mathematically
- Making a generalisation
- Representing

Expect, scaffold, and support your students to use these mathematical practices when sharing their ideas during these starters.

Always, encourage and celebrate all contributions and ideas that are shared from all students. Be ambitious, don't limit your students to small numbers. In Phase Two they need exposure and chances to reason with numbers to at least 1,000,000.

## Important Number Properties within this booklet:

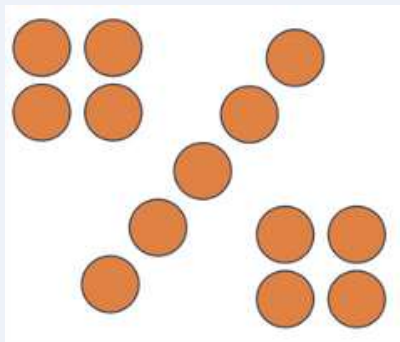
- Inverse: division is the inverse operation of multiplication ( $a \times b = c$  so  $c \div b = a$ ). Multiplication facts give rise to families of facts that use division.
- Identity: when a number is added to 0 the result will be the same, when a number is multiplied or divided by 1 the result will be the same.
- Commutative: we can multiply or add two numbers in any order, and the sum will never change. This does not hold for division or subtraction. E.g.,  $a \times b = b \times a$ ,  $a + b = b + a$ .
- Associative: when adding or multiplying two or more numbers it does not matter what order they are added/multiplied in. E.g.,  $(a \times b) \times c = a \times (b \times c)$
- Distributive: each addend of a sum can be multiplied separately and the product will be the same (e.g.,  $3 \times 17 = 3 \times (10 + 7) = (3 \times 10) + (3 \times 7)$ ).

Most of these starters can be adapted and used as independent tasks as well.

**All students can be successful mathematicians when given the opportunities to succeed.**

# NUMBER - MĀTAURANGA TAU

## Subitising – multiple small sets



### Teacher Notes

Subitising is the ability to recognise the number of objects in a group without needing to count them. This task focusses on noticing multiple small sets at the same time.

### Instructions:

Before putting the image on the screen or board, tell students “I am going to flash something onto the screen for a short period of time, I would like you to see what you notice”

Flash the image onto the screen for 3 seconds

Encourage the students to think about “What did you notice?”

Show image for another 3 seconds

Encourage the students to check their thinking and then share their ideas with a buddy.

Facilitate a whole class discussion discussing, recording and annotating ideas on the diagram.

Highlight the associative property, (if we see 4, 5, and 4 we can add the groups together in any order.

### Curriculum Links

*Partition a pattern of up to 10 objects, instantly recognise the number of objects in each part, and confirm the total number in the pattern using the parts*

### Big Ideas

*Quantity is an attribute of a set of objects and we use numbers (represented by words and symbols) to name specific quantities. A quantity (whole) can be decomposed into different parts, the parts can be composed to form the whole.*

### Mathematical Language

*Numbers 0 – 100, groups of, plus, addition, same as,*

### Suggested Learning Outcomes

*Notice groupings of objects*

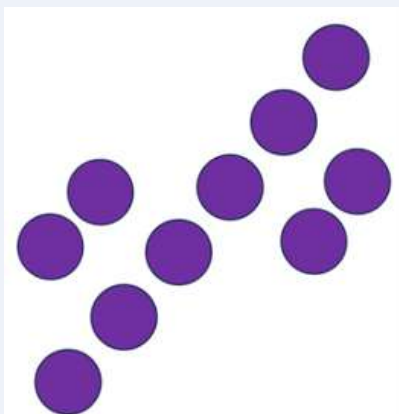
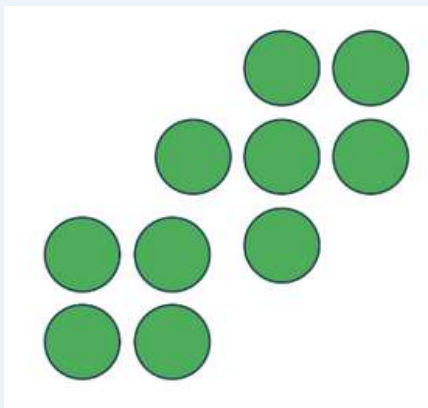
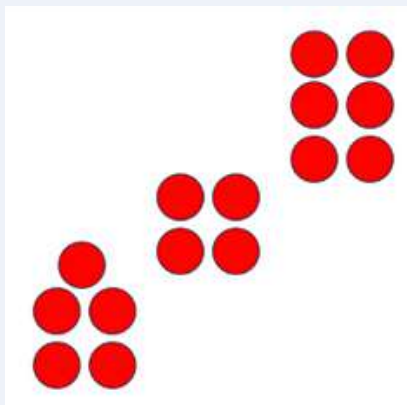
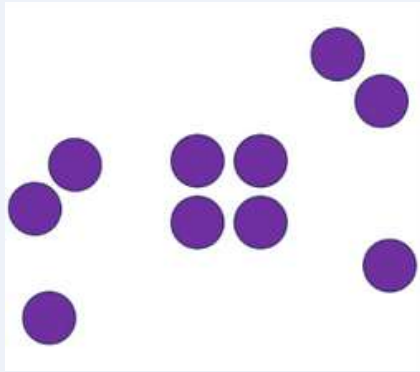
*Join small groups together by adding*

*Instantly know how many objects are in a group to 6*

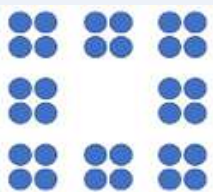


## Other Examples

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## Quick Images - Groups of...



Materials – subitising dot card, or Powerpoint.

### Teacher Notes

Subitising is the ability to recognise the number of objects in a group without needing to count them.

#### Instructions:

Explain to the students you are going to show them an image and they need to think about what they see, and how they see it.

Show the image to the students for 3 seconds. Allow students time to visualize what they saw.

Show the image again for another 3 seconds. Give more time for individual thinking.

Ask students to turn and talk about how many dots they see and why.

Display image again, keeping it displayed this time.



Call on different students to share their thoughts. Record the different ways students saw the image. E.g.,

3 groups of 4, 2 groups of 4, 3 groups of 4



9 groups of 4, but the middle group is missing.

Celebrate the different ways students notice this image. You may wish to explicitly highlight one of the number properties students used. E.g., distributive property  $(3 \times 4) + (2 \times 4) + (3 \times 4) = 8 \times 4$ . Or make links between repeated addition and multiplication  $(4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 8 \times 4)$ .

### Curriculum Links

#### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Use the distributive, commutative, and associative properties

### Big Ideas

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

### Mathematical Language

Multiplication, groups of, commutative property, associative property, distributive property, equal to.

### Suggested Learning Outcomes

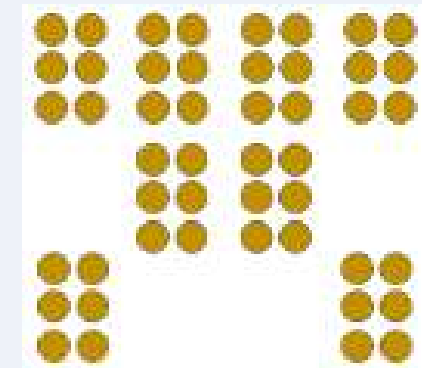
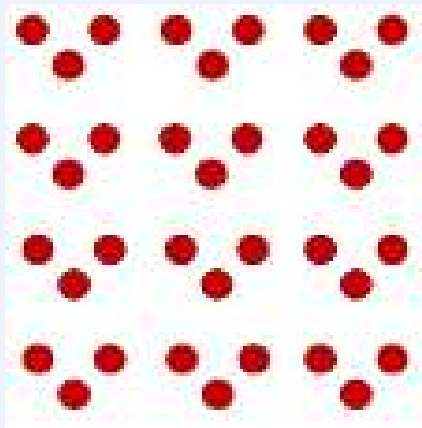
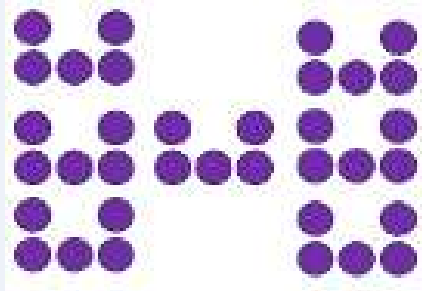
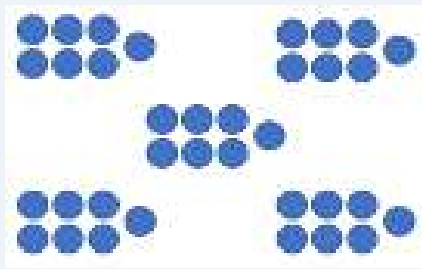
Notice and use groupings to find a total

Recall and apply multiplication facts

Explain how they see an image

## Other Examples

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# Place Value Blocks



What number do these blocks represent?

Materials - use 100s board as visual representation

## Teacher Notes

The purpose of this activity is to support place-value development.

You may wish to use this site to make your own numbers.  
<https://mathigon.org/polypad#number-tiles>

Instructions:

Ask students what number this picture represents?

Give a short time for individual thinking, then ask students to explain their thinking to a buddy and record the number.

Encourage students to use the language of place-value in their explanations. E.g., "I know its 4,526 because I can see 4 thousand cubes, 5 hundreds, 2 tens and 6 ones".

Ask questions that support the nested view of place value. E.g., "How many hundreds are in this thousands block?", "If we had 10 thousand blocks, what number would that be?"

Record the numbers in a variety of ways (expanded form  $4000 + 500 + 20 + 6 = 4526$ , words, digits in a place value house).

## Other Examples

This activity can be repeated multiple times, with a range of numbers and sequences between 0-1,000,000.

This is a great task for an independent activity.

## Curriculum Links

### During Year 4

Identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure

### During Year 5

Identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure

### During Year 6

identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

## Big Ideas

The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

## Mathematical Language

Thousands, hundreds, tens, ones, add, equal, place value

## Suggested Learning Outcomes

Find a total using place-value groupings

Explain the number of thousands, hundreds, tens and ones in any given number

# Place Value to 1,000,000

Millions			Thousands			Ones		
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones
			9	5	1	7	0	9

**What is this number? How do you know?**

## Teacher Notes

Display the image/ or write a number on a blank place-value house. Ask students to turn and tell a partner what the number is and why.

Read the number together. Ensure students are using the correct language “nine hundred & fifty-one thousand, seven hundred & nine”.

Ask a series of questions that focus on the place-value of the numbers. E.g., “What does the 7 represent?”, “What is the value of the tens place?”, “How many ten-thousands are there?”, “What digit is in the hundred-thousand place? What is that digit's place-value?”

Ask students to write the expanded form.  $900,000 + 50,000 + 1,000 + 700 + 9 = 951,709$ .

Extend student's place value understanding by asking questions about the whole number rather than each individual digit, such as “how many hundreds are there in the number 951,709?”

We want students to understand that there are 951 thousands, 9,517 hundreds, 95,170 tens and 951,709 ones. You may want to initially introduce whole number place value understanding with a smaller number.

## Other Examples

Repeat this task multiple times with a variety of numbers.

This is a good practice task for an independent activity.

## Curriculum Links

### During Year 4

Identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure

### During Year 5

Identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure

### During Year 6

identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

## Big Ideas

The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

## Mathematical Language

Thousands, hundreds, tens, ones, add, equal, place value

## Suggested Learning Outcomes

Find a total using place-value groupings

Explain the number of thousands, hundreds, tens and ones in any given number



# Reading and Explaining Numbers to 1,000,000

**45,689**

What is this number?

## Teacher Notes

Ask students, what is this number? Support the students to read the number correctly.

Ask “how could you write or represent this number in different ways?”

Give time for students to work with a partner to record ideas.

Discuss and share the different ideas.

Support students to discuss thousands, hundreds, tens, ones and make links to place, face, and total value.

Link to the place value house as a representation and have this on the wall or whiteboard for students to refer to.

Notice use of place value and the ability to see hundreds as ten tens and tens as ten ones. Draw connections to represent these within place value houses.

To extend the task ask students questions like: “what would the number be if we changed the digit in the tens place to a 5?”, “what would the number be if we add 1000?”, “what would the number be if we moved each digit one place-value to the left/ or right?”

## Other Examples

999,989

104,070

1,000,004

306,060

## Curriculum Links

### During Year 4

Identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure

### During Year 5

Identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure

### During Year 6

identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

## Big Ideas

The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

## Mathematical Language

Millions, thousands, hundreds, tens, ones, add, equal, place value

## Suggested Learning Outcomes

Read numbers up to 1,000,000

Find a total using place-value groupings

Explain the number of millions, thousands, hundreds, tens and ones in any given number

## How many 1's, 10's, 100's 1000, etc

How many?  
10's in 28,107?  
1s in 28,107?  
1,000's in 28,107?  
100s in 28,107?  
10,000s in 28,107?

### Teacher Notes

This starter focuses on the number as a whole rather than individual digit place value.

Present the first question "How many 10's in 28,107?"

Encourage students to turn and talk about what they think and why. Discuss students' ideas. Use materials such as a place-value house or place-value blocks to support student's explanations and understandings.

Reinforce the idea that understanding how many tens are in a number is different to reading the digit in the tens place (place naming). E.g., some students might say there are 0 tens in 28,107 because there is a 0 in the tens place. They may not realise there are 2,810 groups of ten in 28,107

Repeat for the other 4 questions. How many 1's etc. Ensure students understand the place is 10 times bigger than the previous place when we move to the left. E.g. the hundreds place is ten times bigger than the tens place.

### Other Examples

Repeat this activity with other numbers to consolidate this idea.

How many?

10,000s in 1,000,000  
10s in 1,000,000  
1000s in 1,000,000

### Curriculum Links

#### During Year 4

Identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure

#### During Year 5

Identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure

#### During Year 6

Identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

### Big Ideas

The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

### Mathematical Language

Millions, thousands, hundreds, tens, ones, add, equal, place value

### Suggested Learning Outcomes

Read numbers up to 1,000,000

Find a total using place-value groupings

Explain the number of millions, thousands, hundreds, tens and ones in any given number

# Before and After to 1,000,000

Before		After
	801	
	881	
	1391	
	40,801	
	70,931	
	120,391	
	1,000,001	

**What comes before and after each number?**

## Teacher Notes

Students need multiple opportunities to notice and generalise patterns within the structure of our number system.

Instructions:

Ask students “What comes before 801? What comes after 801?” (you may wish to ask 10 or 100 before/after instead).

Record on chart. Repeat for 881, 2391 & 40801 Ask students “What do you notice about the before and after numbers so far? Discuss with a partner.

Complete the chart.

Ask “what claim can we make about these before and after numbers? “How can we test this claim?”.

Support students to notice that if a number has the digit 1 in the ones place, the number before will always have a 0 in the ones place and the number after will always have a 2 in the ones place. The digits in the other places won't change.

## Other Examples

Continue this task with other numbers to consolidate this knowledge.

Also change the value of the before or after number e.g. 20 before or 20 after.

## Curriculum Links

### During Year 4

Identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure

### During Year 5

Identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure

### During Year 6

Identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

## Big Ideas

Numbers, expressions, and measures can be compared by their relative values.

Numerical and algebraic expressions can be compared using greater than, less than, or equal.

## Mathematical Language

Ones, tens, hundreds, thousands, tens of thousands, hundreds of thousands, add, subtract, place value, face value, total value, digit

## Suggested Learning Outcomes

Identify the number before and after any given number

Read numbers to 1,000,000

Notice patterns within numbers

# Order numbers to 1,000,000

299,999      24,160      299,999      659,888      204,160

Order these numbers from biggest to smallest.

## Teacher Notes

Students need multiple opportunities to notice and generalise patterns within the structure of our number system.

Instructions:

Ask the students; What are these numbers? Support the students to read the number correctly.

How can you order these numbers?

Give students an opportunity to work in pairs and record and represent their reasoning.

Allow students opportunities to discuss how the numbers are greater than or less than the others.

Explore concepts, of place, face, and total value. Reinforce that the digit 0 can be used as a place holder. E.g., some students may have the misconception that 299,999.0 is larger than 299,999 because it looks longer, not realising that .0 represents there are no tenths.

## Other Examples

Use combinations of different numbers.

45,876	9,999	460,000
35,999	999	640,000
26,010	99,999	604,000
35,998	99,909	406,000

## Curriculum Links

### During Year 4

Identify, read, write, compare, and order whole numbers up to 10,000, and represent them using base 10 structure

### During Year 5

Identify, read, write, compare, and order whole numbers up to 100,000, and represent them using base 10 structure

### During Year 6

Identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

## Big Ideas

Numbers, expressions, and measures can be compared by their relative values.

Numerical and algebraic expressions can be compared using greater than, less than, or equal.

## Mathematical Language

Ones, tens, hundreds, thousands, tens of thousands, hundreds of thousands, add, subtract, place value, face value, total value, digit

## Suggested Learning Outcomes

Identify the number before and after any given number

Read numbers to 1,000,000

Notice patterns within numbers

# Choral Counting

Count by 5s starting at 251

251	256	261	266
271	276	281	286
291	296	301	306
311	316	321	326
331	336	341	346
A			
		B	
	C		D

**Count in 5's starting at 251**

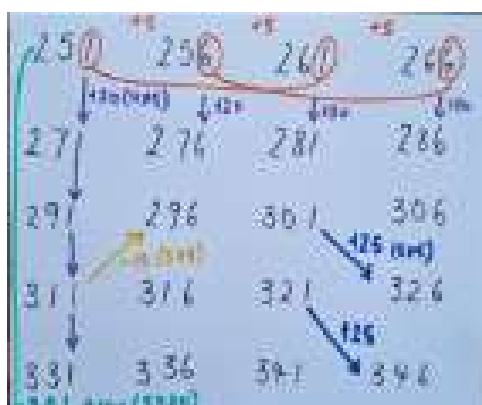
## Teacher Notes

Instructions:

Begin with the first number and count all together, recording the numbers as you go.

When there is a pause or confusion about what comes next, start the count again, going back over what you have already covered.

When the 5<sup>th</sup> line has been filled in, ask students to turn and talk about what they notice. Annotate on the board. E.g.,



Ask “why do you think?” questions to extend thinking. E.g., “why do you think all numbers end with a 1 or a 6?”

Reinforce any mathematical ideas that may appear. E.g., the blue diagonal is +25 because we are adding on +5 five times.

Ask students to predict what numbers will be in squares A, B, C and D by drawing on the patterns they have already noticed in the rows and the columns.

Reminder: curriculum now states students need to skip count in 25s and 50s as well as the basic facts to 10.

## Curriculum Links

**During Years 4, 5 and 6**  
Skip count from any multiple of 100, forwards or backwards in 25s and 50s

Investigate patterns in multiples, using 100s boards or 1,000s books.

Explain patterns and make generalisations or conjectures of recorded choral counts.

## Big Ideas

Skip counting on the number line generates number patterns.

Known elements in a pattern can be used to predict other elements.

## Mathematical Language

Column, row, add, rule, position, pattern, more, less

## Suggested Learning Outcomes

Count in 5's

Notice and make statements about patterns

Predict further positions in a pattern



## Other Examples

Ways to extend the +5 count over a series of days

- Repeat the count of 5 and look for new patterns (students will not see all patterns on the first iteration).
- Repeat starting at a different number.
- Repeat but count backwards. Focus on links to subtraction and division.
- How could we represent this sequence on a graph?
- Make far predictions; what would come at the start of row 20?

Count in  $\frac{3}{5}$

$\frac{3}{5}$	$\frac{6}{5}$	$\frac{9}{5}$	$\frac{12}{5}$	$\frac{15}{5}$
$\frac{18}{5}$	$\frac{21}{5}$	$\frac{24}{5}$	$\frac{27}{5}$	$\frac{30}{5}$
$\frac{33}{5}$	$\frac{36}{5}$	$\frac{39}{5}$	$\frac{42}{5}$	$\frac{45}{5}$
$\frac{48}{5}$				?
		?		

Count in 1.5's starting from 12.5

12.5	14	15.5	17	18.5
20	21.5	23	24.5	26
27.5	29	30.5	32	33.5
35	36.5	38	39.5	41
42.5	44	45.5		?
?		?	?	

Count in 300's starting from 2000.

2000	2300	2600	2900	3200
3500	3800	4100	4400	4700
5000	5300	5600	5900	6200
6500	6800	7100	7400	7700
8000			?	
	?			?

Remember to plan your count before you begin and anticipate the patterns you think the students will notice. Know what mathematical understandings could be reinforced in each count.

See: <https://www.geogebra.org/m/uegm7gu5>

to support planning or

<https://berkeleyeverett.com/choral-counting/> for ideas

# Estimating - How many?



**Estimate how many shoes are outside the mosque to the nearest 10.**

## Teacher Notes

Ask students to turn and tell a buddy an initial estimate or range -reinforce to students that this is not a counting activity. But an estimation activity.

Facilitate a discussion and then ask students to refine their estimate.

Record the different estimations. Ask students to discuss if they think their estimates are reasonable or not.

Ask students to share their approaches for estimating.

Highlight approaches that use groupings, sub-sections and multiplication. E.g., "This section has about 10 shoes, and I think about 7 sections can fit. So maybe its  $10 \times 7 = 70$ ".

Repeat with other interesting photos that require estimating to the nearest 100, 1000 and 10,000.

## Other Examples

Use relevant images to the students - you may like to take photos in nature around them or people at a sports game.

For other ideas use:

<https://estimation180.com>

## Curriculum Links

### During Year 4, 5 and 6

Use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations

Round whole numbers to the nearest thousand, hundred, or ten

## Big Ideas

Numbers, expressions, and measures can be compared by their relative values.

Numerical and algebraic expressions can be compared using greater than, less than, or equal.

## Mathematical Language

Estimate, more, less, total, ten, hundred, thousand, cross-section, groups, multiply

## Suggested Learning Outcomes

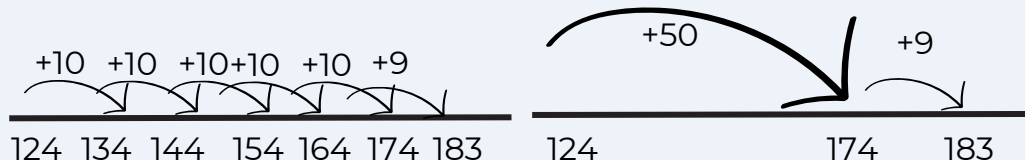
Estimate amounts to the nearest 10, 100, 1000 and 10,000

Recall simple multiplication facts ( $\times 2$ ,  $\times 5$ ,  $\times 10$ )

## What do you notice?

$$124 + 59 = 183$$

What do you notice about these two ways of solving?



Using a number line solve:  $243 + 56 =$

## Teacher Notes

Instructions:

Show the students the two number lines on the board,

Ask the students to turn and talk “what do you notice?”

Facilitate a discussion orienting the students to the idea that both number lines show the same equations in different ways.

- in the first number line 59 has been partitioned into 10s and 9
- the second number line 59 is partitioned into  $50 + 9$
- the answers are the same

Encourage the students to then use their own empty number lines to solve the equation:  $243 + 56 =$

Continue to use this starter to practice and consolidate the use of an empty number line.

You could extend this starter by asking the students to explain each “jump” on the number line into an equation e.g.  $124 + 50 = 174$ ,  $174 + 9 = 183$ .

## Other Examples

Use a variation of numbers to consolidate this knowledge.

Use numbers in the hundreds :  $587 + 131 =$   
-reminding the students to partition the

## Curriculum Links

### During Year 4

Add and subtract two- and three-digit numbers

### During Year 5

Add and subtract whole numbers up to 10,000

### During Year 6

Add and subtract any whole numbers.

## Big Ideas

Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways.

Addition and subtraction and multiplication and division have an inverse relationship.

## Mathematical Language

Ones, tens, hundreds, thousands, add, subtract, place value, face value, total value, digit, addition, subtraction, inverse relationship.

## Suggested Learning Outcomes

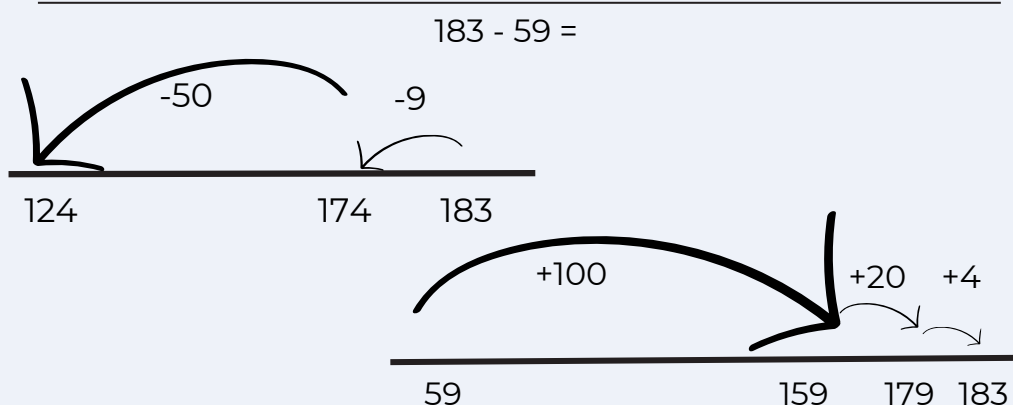
Explain and justify the use of place value to solve subtraction problems.

Explain and justify the use of equivalence and compensation to solve subtraction problems.

Use and justify the inverse relationship between addition and subtraction to solve problems.

Represent equations on an empty number line..

## What do you notice? Inverse



### Teacher Notes

Instructions:

Show the students the two number lines on the board,

Ask the students to turn and talk “what do you notice?”

Facilitate a discussion orienting the students to the idea that both number lines show the same equations in different ways.

- in the first number line 59 is being subtracted from 183, to give an answer of 124.
- the second number line is showing 124 being added to 59 to give 183.

You may need to use colours to represent the relationships between each number line.

Model the equations:  $183 - 59 = \underline{\quad}$  and  $59 + \underline{\quad} = 183$ .

Explain that this is called the inverse if students do not know this language and ask them to look at this equation:

$18 - 5 = 13$  so  $5 + \underline{\quad} = 18$ ?

Continue to consolidate the inverse idea through practice.

### Other Examples

Use a variation of numbers to consolidate this knowledge.

Ask students to solve equations using both subtraction and addition.

$87 - 43 = \underline{\quad}$  and  $43 + \underline{\quad} = 87$

### Curriculum Links

#### During Year 4

Add and subtract two- and three-digit numbers

#### During Year 5

Add and subtract whole numbers up to 10,000

#### During Year 6

Add and subtract any whole numbers.

### Big Ideas

Our number system is based on groupings of ten or base ten. Groupings of ones, tens, hundreds, and thousands can be taken apart in different ways.

Addition and subtraction and multiplication and division have an inverse relationship.

### Mathematical Language

Ones, tens, hundreds, thousands, add, subtract, place value, face value, total value, digit, addition, subtraction, inverse relationship.

### Suggested Learning Outcomes

Explain and justify the use of place value to solve subtraction problems.

Explain and justify the use of equivalence and compensation to solve subtraction problems.

Use and justify the inverse relationship between addition and subtraction to solve problems.

Represent equations on an empty number line.

## Estimating - Are these calculations accurate?

$$367 + 488 =$$

Without calculating - how could we round these numbers to estimate an answer?

### Teacher Notes

Students need to develop an understanding of using estimation to solve equations.

Use this starter once students understand rounding numbers.

Read the question and ask students to discuss with a buddy.

Facilitate a discussion and record the different estimations. Ask students to discuss if they think their estimates are reasonable or not.

Ask students to share their approaches for estimating.

'Once discussed model/represent on the board the use of a number line to prove their estimation.

For other equations, the use of inverse, or commutative properties, distributive property can be introduced to support student estimations and justifications.

### Other Examples

Frankie estimates that  $19 \times 5$  is going to be near 100? Explain why she thinks this?

Frankie knows that  $7 - 3 = 4$ , so she thinks that  $23 - 7 = 14$ .

Do you agree or disagree? Why?

Half of 5625 is about... and why?  
A quarter of 5625 is about...

### Curriculum Links

#### **During Year 4, 5 and 6**

Use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations

### Big Ideas

Numbers, expressions, and measures can be compared by their relative values.

Numerical and algebraic expressions can be compared using greater than, less than, or equal.

### Mathematical Language

Estimate, more, less, close to, total, ten, hundred, thousand, multiply, divide, add, subtract

### Suggested Learning Outcomes

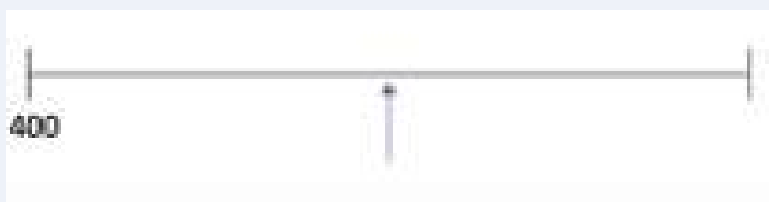
Explain how to round numbers to an appropriate value to make an estimate for a calculation.

Explain reasoning using estimation language such as 'about', 'more or less', and 'close to'.

Explain and justify findings by connecting to estimates and other checking methods.



# Number Line



The arrow is pointing at 500. About where is 427? 540? 590?

## Teacher Notes

### Instructions:

Show the number line and explain that the arrow is pointing to 500.

Ask students to discuss where the numbers are one at a time.

Facilitate a discussion that draws on students explaining and justifying their reasoning.

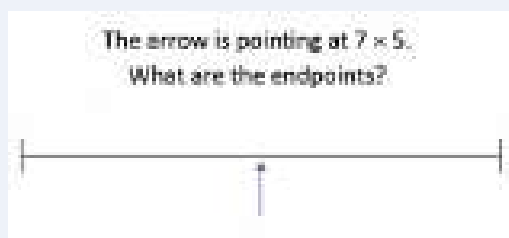
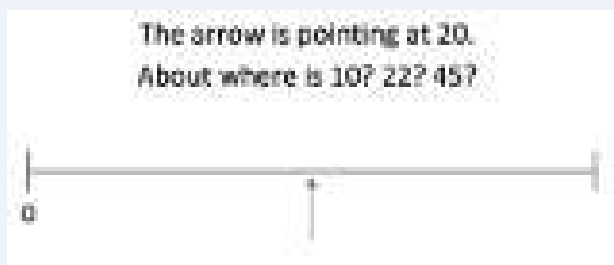
The teacher can annotate and record some benchmark numbers on the number line as the students explain their reasoning and refer to other positions on the number line.

Engage students to debate about whether they agree or disagree with shared reasoning.

## Other Examples

Repeat with a range of number lines, including fractions.

Students may want to make their own number lines and challenge their peers - as an independent activity.



## Curriculum Links

### During Year 4, 5 and 6

Identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

Use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations

Round whole numbers to the nearest thousand, hundred, or ten

## Big Ideas

The set of real numbers is infinite, and each real number can be associated with a unique point on the number line.

## Mathematical Language

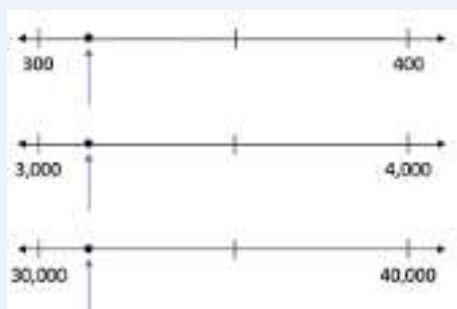
More than, less than, between, approximate, ones, tens, hundred

## Suggested Learning Outcomes

Estimate the position of a number on a number line

Use benchmarks to compare the size of a number

# Triple Number Lines



**What numbers are the arrows pointing to? How do you know?**

## Teacher Notes

### Instructions:

Show the first number line and ask students to discuss what number the arrow might be pointing to and why.

Randomly select students to explain their thinking.

Annotate and record benchmark numbers on the number line as the students explain their reasoning. Expect students to draw on fractional relationships. E.g., “the arrow looks about one third of the way between 300 and 350.” Ask other students “do you agree or disagree, and why?”

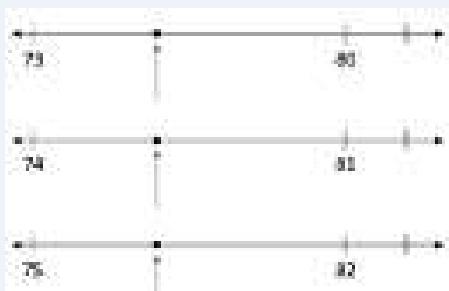
Repeat the process for the other two number lines.

Once all three are completed ask students “What do you notice about all these number lines”

Reinforce the place value relationship between the numbers.

## Other Examples

Repeat with a range of number lines including fractions.



## Curriculum Links

### During Year 4, 5 and 6

Identify, read, write, compare, and order whole numbers up to 1,000,000, and represent them using base 10 structure

Use rounding, estimation, and inverse operations to predict results and to check the reasonableness of calculations

Round whole numbers to the nearest thousand, hundred, or ten

## Big Ideas

The set of real numbers is infinite, and each real number can be associated with a unique point on the number line.

## Mathematical Language

More than, less than, between, hundreds, thousands, ten-thousands, half, quarter, third, sixth

## Suggested Learning Outcomes

Estimate the position of a number on a number line

Use benchmarks to compare the size of a number

## Using known basic facts to solve- using distributive property

Emily says you can solve  $7 \times 9$  by doing  
 $(5 \times 9) + (2 \times 9)$ .

Explain and justify why.

Could you distribute the 7 in any other ways?

### Teacher Notes

#### Instructions:

Give time for students to discuss the claim.

Share ideas as a group. Press students to describe why you can distribute the 7 into 5 and 2.

Encourage students to generalise this idea further by asking could you distribute 7 in any other ways.  
 $(2 \times 9) + (2 \times 9) + (2 \times 9) + (1 \times 9)$  or  $(4 \times 9) + (3 \times 9)$  etc.

Explicitly discuss the idea that if you know and are comfortable using some multiplication or basic facts over others, than use these to solve other equations.

### Other Examples

Repeat with any set of times tables your students need more exposure with. E.g., multiples of 7, multiples of 12

Encourage students to use the multiples that they are most comfortable with to distribute the numbers.

Extend this question by asking can you distribute both factors?

E.g.,  $(5 \times 5) + (2 \times 4) = 7 \times 9$ .

Solve using more challenging numbers – encourage students to discuss how to distribute and less focus on the answer. E.g.,  
 $15 \times 18 = (10 \times 18) + (5 \times 18)$ .

Apply the same starter to a division question e.g.  $92 \div 4 = (80 \div 4) + (12 \div 4)$  or  $(64 \div 4) + (28 \div 4)$  or  $(46 \div 4) + (46 \div 4)$ .

### Curriculum Links

#### During Year 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Use the distributive property

### Big Ideas

There are patterns in the products for multiplication facts

### Mathematical Language

Multiples of numbers to 12, factor, pattern, tens, ones, diagonal, more, less

### Suggested Learning Outcomes

Recall basic multiplication and division facts to 10

Describe a pattern in words or numbers

Identify multiples of 9

# Patterns in Multiples

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

How could you describe the pattern on the hundred-board?  
What do all these numbers have in common?

## Teacher Notes

Students need multiple opportunities to notice and generalise patterns within the structure of our number system. It is not expected they will already “know” these numbers (provide 100’s chart for support).

### Instructions:

Give time for students to discuss the image and the patterns they notice.

Share ideas as a group. Press students to describe the pattern using mathematical language and reasoning.

Students might notice that the digit in the ones place reduces by one each time (9,8,7,6 ...) or the digit in the tens place grows by 1 each time up until 90.

Explicitly connect that this pattern shows the multiples of 9 and we can name these numbers as a position in a pattern. Ask “if 18 is the second multiple of 9, what will be the fifth multiple of 9?”, “If 99 is the eleventh multiple of 9, how could we use this image to predict what the fourteenth multiple of 9 will be?”  
Discuss “how could we use this pattern to know if a number is a multiple of 9 or not?”

## Other Examples

Repeat with any set of multiples your students need more exposure with. E.g., multiples of 7, multiples of 12  
Or provide students with a blank chart, ask them to find the multiples of a given number and discuss what they notice.

## Curriculum Links

### During Year 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply multi-digit whole numbers

## Big Ideas

There are patterns in the products for multiplication facts

## Mathematical Language

Multiples of numbers to 12, factor, pattern, tens, ones, diagonal, more, less

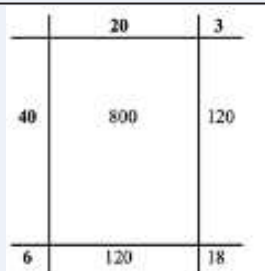
## Suggested Learning Outcomes

Recall basic multiplication and division facts to 10

Describe a pattern in words or numbers

Identify multiples of 9

# Area Model



**What equation is shown in this representation? Explain how you know.**

**Can you show  $337 \times 568$  or  $142 \times 13$  using the area model**

## Teacher Notes

### Instructions:

Display the image and give students time to turn and talk.

Share and discuss their ideas.

Reinforce that in this example students have used place-value partitioning to distribute the numbers into more manageable ones (e.g.  $23 = 20 + 3$ ;  $46 = 40 + 6$ ).

You may also need to recap how we can use known basic facts such as  $4 \times 3 = 12$  to solve  $40 \times 3 = 120$

Record that this model represents application of the distributive property  $(40 \times 20) + (40 \times 3) + (6 \times 20) + (6 \times 3)$ .

Give students time to use the area model to represent other equations such as  $33 \times 56$  or  $142 \times 13$ .

NOTE: Encourage the students to notice the size of each of the squares and the relation to area. An area model is a rectangular grid that can help you solve multiplication problems. The process is similar to how you find the area of a rectangle.

<https://www.youtube.com/watch?v=x8jR7sP45CU>

## Other Examples

What equation does this represent?

	1000	300	20	5
10				
2				

Can you represent  $2449 \times 18$  or  $6021 \times 19$  using this model.

## Curriculum Links

### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply multi-digit whole numbers

Use the distributive, commutative, and associative properties

## Big Ideas

There are patterns in the products for multiplication facts

## Mathematical Language

Area model, factors, multiplication, distributive property

## Suggested Learning Outcomes

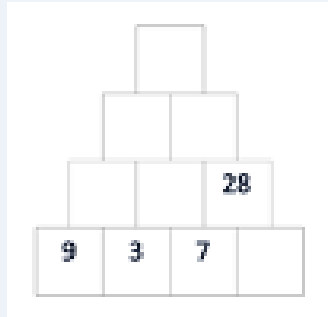
Recall basic multiplication and division facts to 10

Partition numbers into hundreds, tens and ones

Use the distributive property to solve multiplication problems



# Multiplication Pyramid



## Teacher Notes

### Instructions:

Either in pairs, give sufficient time for students to fill in the blank squares. Have access to paper/whiteboard/pen to record representations & times table charts. OR facilitate as a discussion with the whole class.

Expect students to explain and justify as the teacher facilitates discussion to complete the pyramid on the board. Record all student solutions as they are shared as a representation alongside the pyramid.

Make links between the inverse relationship between division and multiplication ( $7 \times ? = 28$ ,  $28 \div 7 = ?$ ).

Connect to number properties. E.g., distributive  
 $(20 \times 20) + (8 \times 20) + (1 \times 20) + (1 \times 8)$


## Other Examples

Make up a variety of multiplication pyramid using different numbers.

These are a great independent activity to consolidate basic facts. Students could make their own for a friend to solve.

## Curriculum Links

### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply multi-digit whole numbers

Use the distributive, commutative, and associative properties

## Big Ideas

There are patterns in the products for multiplication facts

## Mathematical Language

Multiplication, multiply, groups of, factor, product, equals, equivalent, distributive property, commutative property, associative property

## Suggested Learning Outcomes

Recall basic multiplication and division facts to 10

Partition numbers into hundreds, tens and ones

Use the distributive property to solve multiplication problems

# True or False - Factors

**Are these statements true or false?**

**The only factors of 58 are: 2 and 29**

**The only factors of 24 are: 1, 2, 3, 4, 6, 8, 12**

## Teacher Notes

Factor: a positive integer that divides a number exactly with no remainder.

### Instructions:

Ask students to work with a partner to discuss and explain which of the statements are true or false. Ensure they justify their thinking using mathematical reasoning.

Notice students that draw on known multiplication or division facts (or provide access to a timetable chart).

Encourage students to agree or disagree with their peers reasoning explaining why.

Highlight that for any whole number; 1 and itself will always be factors (e.g., 1 and 58 are factors of 58).

## Other Examples

Are these statements true or false?

The only factors of 15 are: 1, 3, 5 and 15.

The only factors of 28 are: 2, 4, 6, 7 and 14.

The only factors of 4 are: 1, 2, and 4

Are the statements true or false?

The only factors of 50 are: 1, 2, 5, 10 and 50

The only factors of 32 are: 1, 2, 4, 8, 16 and 32

The only factors of 62 are: 1 and 31

## Curriculum Links

### During Years 4, 5 and 6

*Recall multiplication facts to  $10 \times 10$  and corresponding division facts*

*Multiply multi-digit whole numbers*

*Use the distributive, commutative, and associative properties*

## Big Ideas

Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways.

## Suggested Learning Outcomes

Find factors of given numbers.

Agree or disagree with reasoning about factors.

## Mathematical Language

*Factor, multiply, divide, whole number, remainder.*

## Factor Table

x				
	16			40
		42	56	
	36			

**What numbers could complete the table? What patterns do you notice?**

### Teacher Notes

Note: the focus is not on completing the table correctly, it is on noticing patterns and relationships to reason mathematically.

### Instructions:

Give students time to work with a partner to explore what number might fit in the missing spaces.

Listen for students who are noticing common factors (e.g., 16 and 36 have three common factors 1, 2 and 4) and use these as a basis for their reasoning.

Highlight and share student ideas that draw on relationships (e.g., “42 and 56 have common factors of 1, 2, 7 and 14. There is a difference of 14 between 42 and 56 so we tried 14 as the multiplier in the third row”).

If required, share an approach for how to find common factors.

### Other Examples

x		10	11	
7		70		84
	99			132
16			176	
	162		198	

x				
	63	72		
				120
				150
	140		180	

### Curriculum Links

#### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

### Big Ideas

Relationships can be described and generalisations made for mathematical situations that have numbers or objects that repeat in predictable ways.

### Suggested Learning Outcomes

Find common factors of numbers

Recall and apply multiplication and division facts

### Mathematical Language

Multiplication, division, factors, equal.

## If... Then...

**If...  $6 \times 3 = 18$  and  $3 \times 6 = 18$**   
**Then...  $18 \div 6 = 3$  and  $18 \div 3 = 6$**

**Could this pattern work for any multiplication sentence?**  
**Explore**

### Teacher Notes

Every multiplication sentence has two related division sentences ( $a \times b = c$  so  $c \div b = a$  and  $c \div a = b$ ).

#### Instructions:

Ask students what they notice about the numbers in the equations. Expect students to justify and reason giving explanations. Highlight student thinking that draws on the inverse relationship.

Give students time to form the own set of related multiplication and division sentences. Share these with the class.

Encourage students to realise that if they know their multiplication facts they can easily solve division by using the inverse.

Note: Students do not need to know how to multiply or know the answers, this task is about noticing the relationship between the sentences.

To extend the task discuss the generalization of  $a \times b = c$  so  $c \div b = a$ .

### Other Examples

If...  $4 \times 9 = 36$  and  $\_\_\_ \times \_\_\_ = \_\_\_$   
Then...  $36 \div 4 = 9$  and  $36 \div 9 = 4$

If...  $15 \times 10 = 150$  and  $10 \times 15 = 150$   
Then...  $150 \div 15 = 10$  and  $\_\_\_ \div \_\_\_ = 15$

If...  $25 \times \_\_\_ = 2500$  and  $\_\_\_ \times 25 = 2500$   
Then...  $2500 \div 25 = 100$  and  $\_\_\_ \div \_\_\_ = \_\_\_$

Use any combinations required to highlight the relationship between multiplication and decimal thinking.

### Curriculum Links

#### During Years 4, 5 and 6

*Recall multiplication facts to  $10 \times 10$  and corresponding division facts*

### Big Ideas

Division facts can be found by thinking about the related multiplication fact.

### Suggested Learning Outcomes

Explain the relationship between multiplication and division

Use multiplication facts to find related division facts

Use the commutative property

### Mathematical Language

*Multiplication, division, inverse, related facts, commutative property*

# Multiplication Strings

$$\begin{aligned}3 \times 4 &= \\30 \times 4 &= \\29 \times 4 &= \end{aligned}$$

## Teacher Notes

These multiplication strings have been designed to encourage students to use known facts and place-value to make solving larger problems easier. Provide access to timetables card if students require them.

### Instructions:

Display the first equation ( $3 \times 4$ ) and give students time to turn and talk about the product and to justify their reasoning.

Expect students to explain and justify as the teacher facilitates discussion about solution strategies.

Display the second equation ( $30 \times 4$ ). Encourage students to look for a relationship between the previous equation in the string and to use this to solve the next equation. E.g.,  $30 \times 4$  is ten times bigger than  $3 \times 4$ . Record all student solutions as they are shared as a representation on the board alongside the number string.

Display the final equation ( $29 \times 4$ ). Ask students "how could use  $3 \times 4$  and  $30 \times 4$  to solve  $29 \times 4$ ?"

Name the number properties if they arise e.g. associative, commutative, distributive.

## Other Examples

$2 \times 9 =$	$6 \times 8 =$	$3 \times 11 =$
$20 \times 9 =$	$6 \times 80 =$	$30 \times 11 =$
$19 \times 9 =$	$6 \times 79 =$	$27 \times 11 =$
$3 \times 50 =$	$2 \times 25 =$	$2 \times 7 =$
$5 \times 50 =$	$4 \times 25 =$	$4 \times 7 =$
$50 \times 50 =$	$8 \times 25 =$	$40 \times 7 =$
$53 \times 50 =$	$10 \times 25 =$	$38 \times 7 =$
$53 \times 49 =$	$16 \times 25 =$	
$3 \times 10 =$	$5 \times 200 =$	$6 \times 20 =$
$3 \times 50 =$	$20 \times 200 =$	$6 \times 100 =$
$3 \times 100 =$	$25 \times 200 =$	$6 \times 120 =$
$3 \times 149 =$	$25 \times 199 =$	$6 \times 119 =$

## Curriculum Links

### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply two-and three-digit numbers

Use the distributive, commutative and associative properties

## Big Ideas

There are arithmetic properties that characterise addition and multiplication as operations. These are the commutative, associative, distributive, and identity properties.

Equations show relationships of equality between parts on either side of the equal sign.

## Suggested Learning Outcomes

Use known facts to solve multiplication problems

Identify relationships between equations

## Mathematical Language

Multiplication, groups of, factor, product, equals, equivalent, distributive property, commutative property, associative property



# Division Strings - Partial Quotients

$$30 \div 3 =$$

$$24 \div 3 =$$

$$54 \div 3$$

## Teacher Notes

These number strings support students to solve problems by breaking the dividend into smaller partial dividends (distributive property). These partial dividends must be divisible by the divisor.

Dividend: the number that will be divided

Divisor: the number the dividend is being divided by

Quotient: product/ answer

## Instructions:

Display the first equation. E.g.,  $30 \div 3$ . Encourage students to turn and talk about the quotient and to justify their reasoning.

Students may draw on known multiplication/division facts or need access to a basic facts chart.

Display the second equation. E.g.,  $24 \div 3$ . Encourage students to turn and talk about the quotient and to justify their reasoning.

Display the final equation. E.g.,  $54 \div 3$ . Encourage students to look for relationships between the previous equations in the string and to use them to solve the next equation.

Record all student solutions as they are shared as a representation on the board alongside the number string.

Reinforce that  $(30 \div 3) + (24 \div 3) = 54 \div 3$

Ask students “why might it be useful to break up a large division equation into smaller ones?” or “why are 30 and 24 useful numbers to choose?” (because they are both factors of the divisor).

## Other Examples

$$200 \div 2 =$$

$$70 \div 2 =$$

$$270 \div 2 =$$

$$360 \div 4 =$$

$$24 \div 4 =$$

$$384 \div 4 =$$

$$40 \div 4 =$$

$$16 \div 4 =$$

$$56 \div 4 =$$

$$16 \div 8 =$$

$$160 \div 8 =$$

$$400 \div 8 =$$

$$80 \div 8 =$$

$$496 \div 8 =$$

$$360 \div 4 =$$

$$24 \div 4 =$$

$$384 \div 4 =$$

## Curriculum Links

### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply two-and three-digit numbers

Use the distributive, commutative and associative properties

## Big Ideas

For a given set of numbers there are relationships that are always true, and these are the rules that govern arithmetic and algebra.

Division algorithms use numerical estimation and the relationship between division and multiplication to find quotients

## Suggested Learning Outcomes

Break numbers into partial dividends

Find multiples and factors

Apply multiplication facts to division problems

## Mathematical Language

divide, division, divisor, dividend, quotient, inverse, multiplication, multiply, groups of, factor, product, equivalent, distributive property

# Fractions in Different Ways



## Teacher Notes

### Instructions:

Ask students to work with a partner to represent the fraction.

Call on a range of students to share their representations. Explicitly discuss how we can represent a fraction in many ways (words, numbers, equations, diagrams, materials etc).

Notice what representations students have used and ask a question that will extend thinking further. E.g., “can we represent this as a decimal fraction?”, “could we represent this on a number line?”, “how do you know three-fifths is equivalent to six-tenths?”

Other questions for large group discussion might be:

- How are the representations the same or different?
- How are the representations connected?

Generalise by choosing a representation then asking “how could we use this representation to show \_\_\_ (change fraction)”

## Other Examples

Show it 3 different ways:  $180 \div 25 =$

Show it 3 different ways: quadrilateral

Show it 3 different ways: 90 degrees

Show it 3 different ways: 420%

## Curriculum Links

### During Year 4, 5 and 6

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:

- compare and order the fractions
- identify when two fractions are equivalent
- represent the fractions in their simplest form

Convert between mixed numbers and improper fractions

## Big Ideas

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

A fraction describes the division of a whole (region, set, segment) into equal parts.

## Suggested Learning Outcomes

Represent a fraction using numbers, pictures and words

Represent a fraction as a decimal and percentage

## Mathematical Language

Whole, half, halves, thirds, sixths, twelfths, fraction, equal, equivalent, mixed numbers, greater than, less than, numerator, denominator.

# Comparing Fractions

A	$\frac{7}{8}$ or $\frac{5}{8}$	$\frac{6}{7}$ or $\frac{5}{7}$	$\frac{3}{12}$ or $\frac{5}{12}$	$\frac{3}{10}$ or $\frac{1}{10}$
B	$\frac{5}{12}$ or $\frac{5}{7}$	$\frac{6}{8}$ or $\frac{6}{11}$	$\frac{2}{5}$ or $\frac{2}{3}$	$\frac{4}{9}$ or $\frac{4}{5}$
C	$\frac{6}{5}$ or $\frac{7}{8}$	$\frac{3}{7}$ or $\frac{5}{8}$	$\frac{10}{11}$ or $\frac{8}{7}$	$\frac{3}{4}$ or $\frac{4}{7}$
D	$\frac{9}{10}$ or $\frac{5}{6}$	$\frac{3}{4}$ or $\frac{2}{3}$	$\frac{6}{7}$ or $\frac{7}{8}$	$\frac{11}{12}$ or $\frac{4}{5}$

Which fraction is bigger? How do you know?

## Teacher Notes

### Instructions:

Each row of this activity (A,B,C,D) supports a different way in which students can reason about the size of fractions.

A - Using same size denominators reasoning – (e.g.,  $\frac{3}{5}$  and  $\frac{4}{5}$ ). Students think about having 3 parts of something and 4 parts of the same thing.

B - Using same numerator reasoning – (e.g.,  $\frac{4}{7}$  and  $\frac{4}{5}$ ). Students think about if the whole is partitioned into 7 parts, the parts will be smaller than if they are partitioned into 5 parts)

C - Using more than/ less than a benchmark reasoning. Some comparisons do not lend themselves to the two above methods of reasoning (e.g.,  $\frac{3}{5}$  and  $\frac{2}{7}$ ).  $\frac{3}{5}$  is more than a  $\frac{1}{2}$  and  $\frac{2}{7}$  is less than a  $\frac{1}{2}$ . 1 can also be used as a benchmark (e.g.,  $\frac{6}{5}$  or  $\frac{7}{8}$ )

D- Closeness to a benchmark. Comparing  $\frac{11}{12}$  and  $\frac{4}{5}$ . Each one is one fractional part away from a whole.  $\frac{11}{12}$  is  $\frac{1}{12}$  away from a whole, so it is closer than  $\frac{1}{5}$ .

### Instructions:

Choose string A,B,C or D. Provide fraction tiles if needed.

Present the first pair of fractions. Ask students to turn and talk about which fraction is bigger and why. Listen for students who are using mathematical reasoning (or have a misconception).

Share back different explanations and reasoning.

Explicitly highlight the type of reasoning students are using (see notes above).

## Curriculum Links

### During Year 4, 5 and 6

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:

- compare and order the fractions
- identify when two fractions are equivalent
- represent the fractions in their simplest form

Convert between mixed numbers and improper fractions

## Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into.

The top number tells how many equal parts are indicated.

## Suggested Learning Outcomes

Compare two fractions

Explain the size of a fraction

## Mathematical Language

Whole, halves, quarters, thirds, sixths, twelfths, eighths, fraction, greater than, less than, numerator, denominator.

## Other Examples

Which Fraction is bigger? Explain your reasoning.

A.	$\frac{15}{8}$ or $\frac{5}{3}$	E.	$\frac{11}{6}$ or $\frac{21}{11}$
B.	$\frac{8}{7}$ or $\frac{3}{7}$	F.	$\frac{7}{6}$ or $\frac{13}{8}$
C.	$\frac{5}{4}$ or $\frac{12}{7}$	G.	$\frac{9}{5}$ or $\frac{9}{11}$
D.	$\frac{15}{8}$ or $\frac{15}{6}$	H.	$\frac{2}{12}$ or $\frac{13}{12}$

Which Fraction is bigger? Explain your reasoning.

A.	$\frac{3}{5}$ or $\frac{4}{5}$	E.	$\frac{6}{7}$ or $\frac{2}{3}$
B.	$\frac{4}{7}$ or $\frac{4}{5}$	F.	$\frac{7}{11}$ or $\frac{5}{11}$
C.	$\frac{3}{5}$ or $\frac{2}{7}$	G.	$\frac{3}{7}$ or $\frac{5}{11}$
D.	$\frac{11}{12}$ or $\frac{4}{5}$	H.	$\frac{8}{12}$ or $\frac{8}{7}$

## True or False - Convert Improper Fractions

$$2\frac{7}{3} = \frac{7}{3}$$

**Materials - Fraction Tiles**

### Teacher Notes

Ask students to name some proper and mixed fractions. Record it on the whiteboard.

Write the mixed fraction and improper fraction on the whiteboard and ask students to explain and justify if it's true or false.

When the students share back, record their justification including representation on the whiteboard.

Highlight to the students that an improper fraction has a numerator greater than the denominator.

Draw this representation on the whiteboard and count the thirds.



Write the fraction as a mixed fraction by counting how many wholes and parts.

### Other Examples

True or False:

1.  =  $\frac{12}{10}$

2.  $\frac{11}{4} = 2\frac{3}{4}$

3.  $\frac{17}{5} = \frac{3}{5}$

### Curriculum Links

#### During Year 4, 5 and 6

For fractions with denominators of 2, 3, 4, 5, 6, 8, 10, 12, or 100:

- compare and order the fractions
- identify when two fractions are equivalent
- represent the fractions in their simplest form

Convert between mixed numbers and improper fractions using number lines

### Big Ideas

A fraction describes the division of a whole (region, set, segment) into equal parts.

The bottom number in a fraction tells how many equal parts the whole or unit is divided into.

The top number tells how many equal parts are indicated. A fraction is relative to the size of the whole or unit.

### Suggested Learning Outcomes

Convert improper fraction to mixed fraction.

Explain and justify their thinking.

### Mathematical Language

Whole, fraction, improper fraction, mixed fraction, denominator, numerator

# Round to the Nearest...

## Round to the Nearest...

	whole number	...tenth	...hundreth
43.286			
126.987			
290.011			

## How do you know?

### Teacher Notes

Give an opportunity to discuss and justify with someone else before they share their ideas.

Have a place value house for whole and decimal numbers on the wall or give to students to use if needed.

### Other Examples

Here are some other examples you can use on other days you can explore one of these numbers or get them to try one of the three to justify.

Day 2	3.231	93.149	33.645
Day 3	560.297	5610.999	301.732
Day 4	1299.777	2003.182	2110.618
Day 5	2999.847	7165.487	4999.956

### Curriculum Links

#### During Year 4, 5 and 6

Identify, read, write, and represent fractions, decimals (to two places), and related percentages

Round whole numbers to a specified power of 10, and round tenths and hundredths to the nearest whole number or one decimal place

### Big Ideas

Decimals are a set of fractions that have powers of 10 as their denominators (e.g.,  $\frac{170}{10}$  or  $\frac{1070}{100}$ ) and that can be written as numbers using a decimal point (e.g., 0.7 or 0.07).

A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line

### Suggested Learning Outcomes

Round decimals to the nearest whole number, tenth or hundredth.

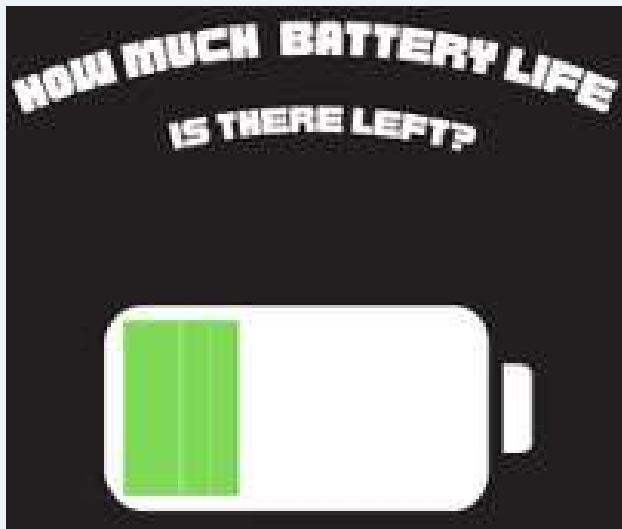
Justify their reasoning.

### Mathematical Language

Decimals, whole number, place value, tenths, hundredths.



# Battery Percentages



## Teacher Notes

1 Display the picture and give time for students to find ways to describe/represent how much battery life is left.

Share student's ideas and representations. Expect a clear explanation that draws on understandings about fractions, decimals or percentages.

Encourage students to see connections between a variety of representations. Press for justification of why they are all describing the same amount. For example, 50% is the same as  $\frac{1}{2}$  because 50% is half of 100% and 100% is one whole battery.

Support students to develop the understanding that percentages are a type of equivalent fraction with a denominator of 100.

To extend the task link to bonds to 100. E.g., ask students "if 30% of the battery is full, what percent is empty?"  $30\% + ? = 100\%$  or  $100\% - ? = 30\%$

## Other Examples

You can repeat this activity using objects from around the classroom. E.g., partially fill a jar with objects/ water and estimate what percentage is full, and what percentage is empty.

## Curriculum Links

**During Years 4, 5 and 6**  
Identify, read, write, and represent fractions, decimals (to two places), and related percentages.

## Big Ideas

A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.

Percent is relative to the size of the whole.

## Suggested Learning Outcomes

*Estimate a percentage of a whole object*

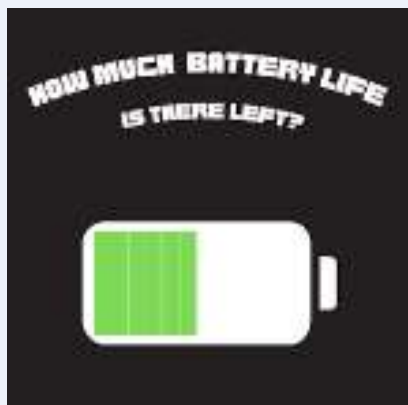
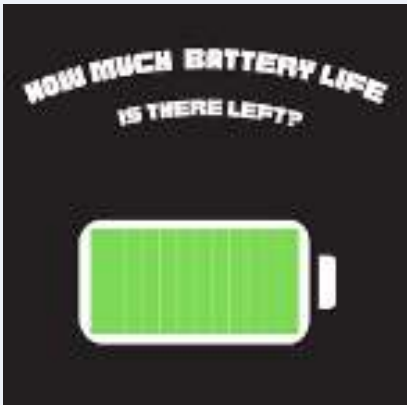
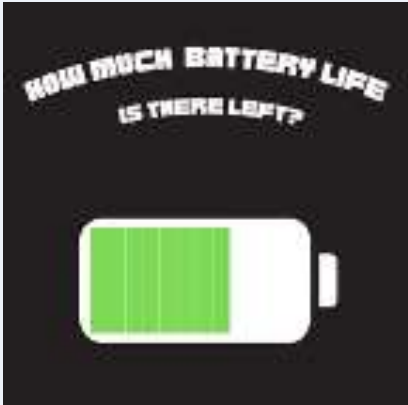
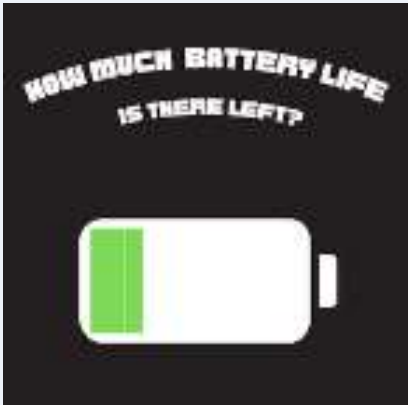
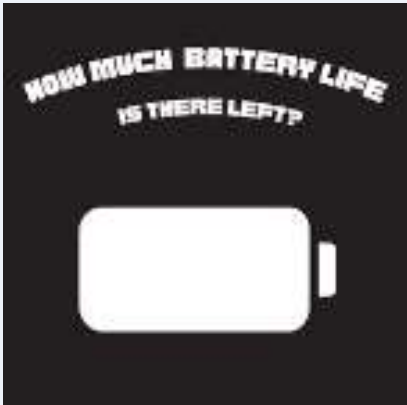
*Convert simple fractions to percentages*

*Notice connections between representations*

## Mathematical Language

*Whole, percentage, benchmark, greater than, less than, equivalent, numerator, denominator, fraction names (e.g., thirds)*

Other Examples



# Missing Decimal Numbers

.01	.02	.03	.04	.05	.06	.07	.08	.09	.10
.11									
				.25					
				.35					
								.50	
.51	.52								.60
			.74						
								.89	.90
								.99	1.00

What are the missing numbers? How can you prove it? What patterns do you see? What would the decimals be as an equivalent fraction? What about as a percentage?

## Teacher Notes

Present the image and allow some time for students to look and think about what they see. Ask students to turn and talk/ record the missing decimal numbers and explain their thinking.

Select students to share a missing number. Expect students to provide a reason. E.g., "I think its 0.22 because ..."

Facilitate discussion by asking students if they agree or disagree with the reasoning shared. Repeat for all the missing numbers.

Ask students to discuss the patterns they can see in the image. What does decimal counting have in common with whole number counting?

Repeat the task, but this time ask students to record the missing numbers as either a percentage or an equivalent fraction.

Discuss and record the equivalent fractions and percentages.

## Curriculum Links

### During Years 4, 5 and 6

Identify, read, write, and represent fractions, decimals (to two places), and related percentages.

## Big Ideas

Decimal place value is an extension of whole number place value.

A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.

## Suggested Learning Outcomes

Read and write decimal numbers to hundredths

Notice patterns in the number system

Convert decimal fractions to percentages

## Mathematical Language

Place value, base ten, tenths, hundredths, decimal, fraction, percentage, equivalent fraction

## Other Examples

.001	.002	.003	.004	.005	.006	.007	.008	.009	.010
.011									.020
				.005					.010
				.005					.010
.011	.012								.020
.011	.012								.020
.011	.012	.013	.014		.016				.020
								.009	.010
								.009	.010

# Decimal Place Value Partitioning



## Teacher Notes

The focus of this task is to practice naming and adding decimals in the tenths and hundredths place (using place-value partitioning).

### Instructions:

Ask students to turn and talk about what values should be written on the empty ones, tenths and hundredths lines.

Expect explanations that use correct place-value language for tenths and hundredths, "there are 5 tenths because..."

Allow time for students to solve the equation, either mentally or written. Share back some responses.

Record the different students' representations and ideas for adding decimals.

Ensure students explain their ideas using place-value language, "4 hundredths + 2 hundredths = 6 hundredths".

Reinforce the base 10 understanding that 10 hundredths are equivalent to 1 tenth. 10 tenths are equal to 1 one.

Model on materials if needed.

## Other Examples

$$5.65 + 7.89 =$$

$$3.66 + 6.44 =$$

$$6.03 + 5.09 =$$

## Curriculum Links

### During Year 4

Add and subtract decimals to one decimal place (e.g.,  $1.3 + 0.2 = 1.5$ )

### During Year 5

Add and subtract whole numbers and decimals to two decimal places (e.g.,  $32.55 - 21.21 = 11.34$ )

### During Year 6

Add and subtract whole numbers and decimals to two decimal places (e.g.,  $250.11 + 135.29 = 385.4$ )

## Big Ideas

A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.

The effects of operations for addition and subtraction with fractions and decimals are the same as those with whole numbers.

## Suggested Learning Outcomes

Solve problems involving decimal by adding or subtracting and explain and justify the solution.

Represent reasoning to explain and justify place value involving decimal numbers.

## Mathematical Language

Percent, percentage, whole, fraction, fractional number, decimal number, rational number, equal, equivalent.

# Decimal Addition - Missing Addends

\_\_\_ + \_\_\_ = 4.024

What could the missing addends be in this sum?

## Teacher Notes

### Instructions:

Recap that an addend is a number that is added to another one.

Give students sufficient time to record some possible solutions.

Call on students to explain possible solutions and ensure correct place-value language is used. E.g., 3 wholes & 2 hundredths + 1 whole and 4 thousandths.

Push for students to provide reasoning and justification about why their two missing addends are equal to 4.024 (they may need access to materials to prove this).

Facilitate discussion by asking students if they agree or disagree with the reasoning shared.

Refer to the place value house throughout the discussion to make connections to the value of the digits and to highlight the base ten number system.

### Other Examples

___ + ___ = 12.632	6.71 = ___ + ___	___ + ___ = 7.985
10.10 = ___ + ___	___ + ___ = 0.406	___ + ___ = 0.030

## Curriculum Links

### During Year 4

Add and subtract decimals to one decimal place (e.g.,  $1.3 + 0.2 = 1.5$ )

### During Year 5

Add and subtract whole numbers and decimals to two decimal places (e.g.,  $32.55 - 21.21 = 11.34$ )

### During Year 6

Add and subtract whole numbers and decimals to two decimal places (e.g.,  $250.11 + 135.29 = 385.4$ )

## Big Ideas

Decimals are a set of fractions that have powers of 10 as their denominators and that can be written as numbers using a decimal point. Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

## Suggested Learning Outcomes

Add tenths and hundredths

Solve open-ended addition problems

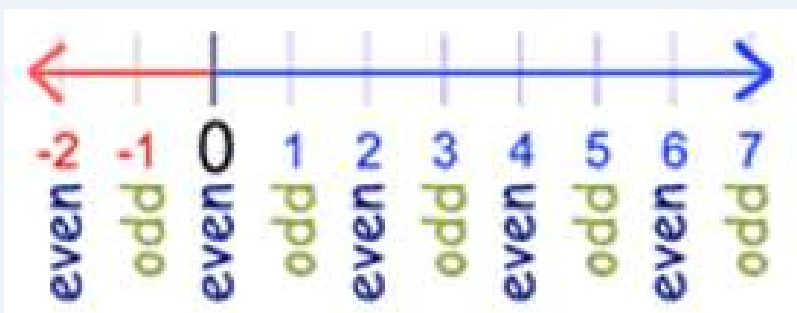
Justify and explain their thinking.

## Mathematical Language

Place value, base ten, tenths, hundredths, thousandths, decimal, equals, equivalent, addition, addend, sum

## Make Statements About Odd and Even Numbers

What do you notice about this number line?



Choose an even number 2 to 6 and make a statement explaining why it is an even number?

### Teacher Notes

The purpose of this activity is to explore a pattern within our number system: odd and even numbers.

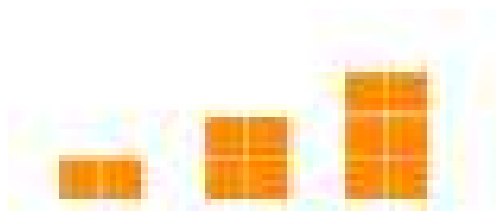
### Instructions:

Ask students to look at the number line and choose an even number between 2 – 6. Students may notice negative numbers, discuss when they may have seen these types of numbers (eg. Temperature in winter, the dial on their freezer, bank accounts...). Also that 0 is an even number. Discuss the reason after they have explored their numbers and coming up with their own statements (zero is an even number because it 'fits into' the pattern of even numbers and when zero is added or subtracted from an even number, the result is an even number.)

Give a short time for individual thinking, then ask students to explain their thinking to a buddy.

Listen for students saying things: 'they are equal or have the same size groups', 'you keep on adding 2', 'it's +2 each time', 'they go in pairs', 'they match and have partners', 'they're called even because there's none left over', 'it's kind of fair'.

If students are not noticing or unsure that even numbers can be divided into two equal groups with no remainders. Ask them to choose one colour unifix cubes and make a cube model of the even numbers to 10.



### Curriculum Links

#### During Years 4, 5 and 6

Recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule for the pattern in words, and make conjectures about further elements or terms in the pattern

### Big Ideas

The base ten numeration system is a scheme for recording numbers using digits 0-9, groups of ten, and place value.

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

### Suggested Learning Outcomes

Identify even and odd numbers

Independently investigate, recognise and report on the patterns and characteristics of even numbers and of odd numbers.

State generalisations about the addition and subtraction of even numbers and of odd numbers.

### Mathematical Language

Numbers, add, subtract, odd, even, patterns, investigate, same as, equal.



## Teacher Notes continued

Encourage the use of a variety of representations eg. Drawings, use of everyday objects, materials to prove if their number is even or not.

Encourage students to see the links between their doubling and halving knowledge, 2 times tables and the recognise that this is a pattern that grows by +2 each time.

If no student comes up with the statement write this on the board When even numbers are added together the sum (answer) is always an even number. Ask students to check if this generalisation is true for the even numbers they chose?

Go beyond the numbers on the number line by using the hundreds board to prove this generalisation. Check if they notice all even numbers end in 0,2,4,6,8.

Repeat question using odd numbers.

Ask students with their buddy to choose one generalisation and use materials to prove this to the whole group?

Even (+ or -) even = even number

Even (+ or - ) odd = odd number

Odd (+ or - ) odd = even number

Odd ( + or - ) even = odd number

Or use these sentences depending on your students.

When one odd number is added to another odd number the sum is an even number.

When an odd number of odd numbers are added together, the result is an odd number.

When an even number of odd numbers are added, the result is an even number.

When one odd number is subtracted from an odd number the result is an even number.

### Other Examples

Choose an odd number and make a statement why you think it is an odd number?

Choose one generalisation and using materials prove this to your group?

Even (+ or -) even = even number

Even (+ or - ) odd = odd number

Odd (+ or - ) odd = even number

Odd ( + or - ) even = odd number

# Where is the Maths?



**What maths can you see in this photo?**

**What maths question could we ask?**

(Note: use photos that will be engaging for your local community)

## Teacher Notes

This activity can cover all strands of the mathematics curriculum (number, measurement, algebra, space, statistics & probability).

### Instructions:

Display the picture and ask students to discuss in pairs what mathematics they see in the picture.

Share and collate all the ideas. Notice what students identify and ask a question that will extend their thinking. E.g., “where might time/area/money be in this photo?”

Ask “what maths question could we ask about this picture?”.

Give students time to work with a partner to record questions.

Collect and share ideas.

To extend the activity, ask students to estimate (with reasoning) the answer to one of their questions.

## Other Examples

Use any photo, artwork or short video relatable to your students.

## Curriculum Links

### During Years 4, 5 and 6

*Pose a question for investigation*

*Make connections with ideas in other learning areas and in familiar cultural, linguistic, and historical contexts.*

## Big Ideas

*The world is full of patterns and structures that we use mathematics and statistics to understand.*

*Mathematical practices are central to learning and doing mathematics.*

## Suggested Learning Outcomes

*Form a mathematics question*

*Identify mathematics in everyday contexts*

## Mathematical Language

*Question, length, time, angle, amount, money, height, area ...*

# Make Amounts of Money Using Dollars and Cents

Using coins	
\$0.60	\$2.80
\$10.50	\$51.10

In what different ways can we show these amounts of money using dollars and cents?

## Teacher Notes

In this activity, students get to figure out as many different way to make the money totals, firstly using coins only (Including \$1 & \$2 coins), then using dollars including (\$1 & \$2 coins), followed by using combinations of coins and notes to make the totals.

Provide the students with time to reason why they used these combinations for their totals and justify why they used these combinations.

Facilitate shared discussions making comparatives of the variable combinations the students used.

Questions to support discussion:  
Why did you use those combinations?  
What did you notice as you were working with the combinations?  
Did you notice any patterns emerge from this activity?

## Other Examples

Using notes + coins	
\$8	\$17
\$149	\$1,089

Using notes + coins	
\$7.40	\$19.80
\$153.70	\$2,194.20

## Curriculum Links

**During Year 4**  
Make amounts of money using dollars and cents (e.g., to make 3 dollars and 70 cents)

**During Year 5**  
Represent money values in multiple ways using notes and coins

**During Year 6**  
Solve problems involving purchases (e.g., ensuring they have enough money)

## Big Ideas

For most money amounts, there are different, but finite combinations of currency that show the same amount; the number of coins in two sets does not necessarily indicate which of two sets has the greater value.

## Suggested Learning Outcomes

Make groupings of money.  
Use cents and dollars to make amounts of money.

## Mathematical Language

Ones, tens, hundreds, thousands, add, subtract, place value, face value, total value, digit, addition, subtraction, inverse relationship, dollar, cents, notes, coins

# Developing Basic Facts Recall

×	2	3	4	5	10
5					
10					
2					
4					
3					

×	3	5	4	2	10
2					
5					
10					
4					
3					

## Curriculum Links

**During Years 4, 5 and 6**  
Recall multiplication facts to  $10 \times 10$  and corresponding division facts

## Teacher Notes

### Year 4:

Give students a grid for the 2s, 3s, 5s, and 10s and ask students to solve them and record their time (in a non-public way).

### Year 5 and 6:

Give students a grid for times tables up to the 10s and ask students to solve them and record their time (in a non-public way).

Use the following website to generate different grids:  
<http://www.mental-arithmetic.co.uk/multiplication-grids-pdf-generator.htm>

This activity can be used throughout the year as a starter or independent activity to support the development and fluency of times tables.

## Big Ideas

Numbers can be composed and decomposed in different ways by using patterns.

## Suggested Learning Outcomes

Read and write multiplication division equations

Explain the inverse relationship between multiplication and division

## Mathematical Language

Number words, multiply, divide, equals, equality, balance, commutative property, pattern, inverse, family of facts

# Number Bonds

**Part 1:** Look at the following number sentences.

What do you notice?

What other number sentences could fit with this set?

$$9 \times 5 = 45$$

$$45 \div 9 = 5$$

$$45 = 5 \times 9$$

$$45 = 9 \times 5$$

**Part 2:** What number sentences should go with  $9 \times 50 = 450$

## Teacher Notes

Key concepts: equality, commutative property of multiplication, inverse relationship between multiplication and division.

### Instructions:

Present the first set of number sentences (Part 1) to students and ask “What do you notice? What other equations could fit with this set?”

Students turn and talk to a partner. Listen to/ and record student responses. Press for explanations or reasoning that draws on patterning, equality or relationships.

Other equations that go with the set:  $5 \times 4 = 9$ ,  $45 \div 5 = 9$ ,  $9 = 45 \div 5$ ,  $5 = 45 \div 9$

Present  $9 \times 50 = 450$ . Students to record other number sentences that follow this pattern.

Discuss that if we know  $9 \times 5 = 45$ , we can use this to solve  $9 \times 50 = 450$  (50 is 10x bigger than 5).

Highlight which equations apply the commutative property of multiplication, and which use inverse operations.

## Curriculum Links

### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply two- and three-digit whole numbers

Divide whole numbers by one- or two-digit divisors

## Big Ideas

Numbers can be composed and decomposed in different ways by using patterns.

The equal sign is relational; it shows that the two sides of an equation are the same.

## Suggested Learning Outcomes

Recognise expressions that are equal in value.

Read and write multiplication division equations

Explain the inverse relationship between multiplication and division

## Follow and apply a pattern Mathematical Language

Number words, multiply, divide, equals, equality, balance, commutative property, pattern, inverse, family of facts

## Other Examples

### Teen facts (add/sub)

$$\begin{aligned}10 + 3 &= 13 \\13 - 10 &= 3 \\13 &= 10 + 3 \\13 - 3 &= 10\end{aligned}$$

$$10 + 7 = 17 \dots$$

### Basic Facts to 10

$$\begin{aligned}5 \times 10 &= 50 \\50 \div 10 &= 5 \\10 &= 50 \div 5 \\10 \times 5 &= 50\end{aligned}$$

$$7 \times 10 = 70 \dots$$

### 2-digit x 3-digit

$$\begin{aligned}50 \times 152 &= 7600 \\7600 &= 152 \times 50 \\7600 \div 50 &= 152 \\50 &= 7600 \div 152 \\7600 &= 50 \times 152\end{aligned}$$

$$75 \times 101 = 7575 \dots$$

### 2-digit x 2-digit

$$\begin{aligned}22 \times 12 &= 264 \\264 \div 22 &= 12 \\12 &= 264 \div 22 \\264 &= 12 \times 22\end{aligned}$$

$$32 \times 12 = 384 \dots$$

### Hundreds

$$\begin{aligned}4 \times 600 &= 2400 \\2400 \div 4 &= 600 \\2400 &= 4 \times 600 \\600 &= 2400 \div 4\end{aligned}$$

$$4 \times 800 = 3200 \dots$$

### 3-digit x 3-digit

$$\begin{aligned}500 \times 200 &= 100,000 \\100,000 &= 200 \times 500 \\100,000 \div 200 &= 500 \\200 &= 100,000 \div 500 \\200 \times 500 &= 100,000\end{aligned}$$

$$300 \times 700 = 210,000$$



# Doubling and Halving

$$240 \div 8 =$$

$$120 \div 4 =$$

$$60 \div 2 =$$

$$\_\_\_ \div 1 =$$

What do you notice about these numbers? Can you complete the pattern?

## Teacher Notes

Doubling and halving involves using proportional adjustment to make multiplication problems easier to solve.

## Instructions

Show students the multiplication equations. Ask “what do you notice about these numbers?”

Give students time to turn and talk to a partner. Facilitate a discussion based on student's ideas. They might notice that for  $1 \times 60$  &  $2 \times 30$  that 2 is double 1 and 30 is half of 60.

Ask students to complete the pattern. Discuss “why do we always need to double one side and half the other side?”

Repeat as above for the division equations. Students might notice 120 is half of 240 and 4 is half of 8. Ask “why do we need to half both sides”

Record the rule for multiplication and the rule for division on the board. Ask “why do you think these rules are different?”

## Other Examples

$$1 \times 60 =$$

$$2 \times 30 =$$

$$4 \times 15 =$$

$$8 \times \_\_\_ =$$

$$3 \times 100 =$$

$$6 \times 50 =$$

$$12 \times \_\_\_ =$$

$$36 \div 6 =$$

$$18 \div 3 =$$

$$\_\_\_ \div 1.5 =$$

$$1 \times 40 =$$

$$2 \times 20 =$$

$$4 \times 10 =$$

$$8 \times \_\_\_ =$$

## Curriculum Links

### During Years 4, 5 and 6

Recall multiplication facts to  $10 \times 10$  and corresponding division facts

Multiply two- and three-digit whole numbers

Divide whole numbers by one- or two-digit divisors

## Big Ideas

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

## Suggested Learning Outcomes

Explain and justify relationships between numbers in an equation

Use doubling and halving

## Mathematical Language

Double, half, divide, multiply, proportional

# Balancing Equations

$$2 \times \underline{\quad} = 4 \times \underline{\quad}$$

How can we balance this equation?

$$2 \times \underline{\quad} = 4 \times 16$$

What must be the missing factor?

Which side would you rather solve?

How might this help you with other problems?

## Teacher Notes

### Instructions:

Reveal the first equation ( $2 \times ? = 4 \times ?$ ). Ask students “what numbers might we put in the space to balance this equation?”

Encourage students to turn and talk about the products and to justify their reasoning.

Record all student solutions as they are shared as equations on the whiteboard. Discuss that there are multiple ways to balance this equation (e.g.,  $2 \times 60 = 4 \times 30$ ,  $2 \times 5 = 4 \times 2.5$ )

Highlight solutions that draw on noticing a relationship between the left and right side of the equation (as opposed to calculating answers through trial and error).

If students describe the equation  $2 \times 4 = 4 \times 2$  then discuss the commutative property of multiplication.

Reveal the second equation  $2 \times ? = 4 \times 16$ . Give time for students to discuss what the missing factor is.

Use the questions as a discussion prompt to unpack the doubling and halving (proportional adjustment) relationship as the associative property of multiplication.

## Other Examples

$$6 \times \underline{\quad} = 3 \times \underline{\quad}$$
$$6 \times \underline{\quad} = 3 \times 24$$

$$5 \times \underline{\quad} = 10 \times \underline{\quad}$$
$$6 \times 50 = 10 \times \underline{\quad}$$

$$8 \times \underline{\quad} = 4 \times \underline{\quad}$$
$$8 \times \underline{\quad} = 4 \times 19$$

## Curriculum Links

### During Year 4, 5 and 6

Form and solve true or false number sentences and open number sentences involving all four operations, using an understanding of equality or inequality.

## Big Ideas

There are arithmetic properties that characterise addition and multiplication as operations. Equations show relationships of equality between parts on either side of the equal sign.

## Suggested Learning Outcomes

Recall and apply multiplication facts

Balance equations by finding relationships

Explain the equals sign (=) represents balance

## Mathematical Language

Multiplication, multiply, groups of, factor, product, equals, commutative property, associative property, distributive property

## Equal or Not Equal?

$$40 \times 3 = 40 + 40 + 40$$

$$100 \div 5 = 25$$

$$18 \times 3 = (10 \times 3) + (8 \times 3)$$

$$1.010 = 1.09 + 0.01$$

$$2^5 = 2 \times 2 \times 2 \times 2$$

$$200 \times 4 = 4 \times 200$$

Are these equations equal = or unequal  $\neq$  ?

Be ready to explain your reasoning.

### Teacher Notes

Introduce students to the notation  $\neq$  if a new concept. Discuss that we can use  $\neq$  to show if an equation is not equal or balanced.

Orientate students to the table and ensure they understand they use choose = or  $\neq$  to complete the equations.

Allow students time to work and discuss with a partner.

Choose students to share their solution to each equation. Expect a valid mathematical reason as to why it is equal or unequal.

Ask students if they agree or disagree with the reasoning.

These equations require understandings such as distributive property, exponents, adding decimals. Notice if students have any misconceptions and address these.

### Other Examples

Ask students to write their own set of equations that include both equal and unequal examples.

### Curriculum Links

#### **During Year 4, 5 and 6**

Add and subtract whole numbers and decimals to two places

Multiply two- and three-digit whole numbers

Divide whole numbers by one- or two-digit divisors

The equal (=) and inequality (<, >) signs show relationships.

### Big Ideas

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

### Suggested Learning Outcomes

Explain how an equation is equal or unequal

Use the symbols = or  $\neq$  to show equality and inequality.

Give a mathematical reason

### Mathematical Language

Equal, unequal, equality, inequality

# What Could the Equation Be?

The Number is **3006**

What could the equation be?

## Teacher Notes

This activity will encourage students to think flexibly about number, and to explore some of the infinite ways a number can be represented. Any previous number/algebra learning can be reinforced in this task if highlighted.

### Instructions

Allow students a short amount of time to form ideas with a partner.

Gather and record all the different ideas.

Explicitly highlight any ideas that support previous learning (e.g., associative property, decimals, place-value, equality) or are outside the box (  $-119 = 3006$  or  $3x = 3006$  )

Ask a question that will extend student thinking. E.g., “Could we use thousandths to make an equation that equals 3006?”, “is there another way we could use exponents?”

Continue to add to possible solutions over several days or assign as an independent task.

## Other Examples

Whole numbers: 31, 175, 5920, 14 002, 429 655, 2 000 000 ...

Decimal numbers: 2.5, 45.1, 800.5, 13 670.3, 1 000 000.7 ...

## Curriculum Links

### **During Year 4, 5 and 6**

Add and subtract whole numbers and decimals to two places

Multiply two- and three-digit whole numbers

Divide whole numbers by one- or two-digit divisors

The equal (=) and inequality (<, >) signs show relationships.

## Big Ideas

Any number, measure, numerical expression, algebraic expression, or equation can be represented in an infinite number of ways that have the same value.

## Suggested Learning Outcomes

Divide by grouping and using number patterns

Partition a two digit number into tens and one to divide

Use known division facts to work out the quotient of unknown facts.

## Mathematical Language

equation, place value, partition, tens, one, divide, division, divisor, dividend, quotient.

# Number Sequences - Same and Different

Look at these two number sequences.  
What is the same and what is different?

- A) ...19.5, 19, 18.5, 18, 17.5, 17, 16.5, 16, 15.5, 15, 14.5 ...  
B) ...13.5, 14.5, 15.5, 16.5, 17.5, 18.5, 19.5, 20.5 ...

## Teacher Notes

### Instructions

Give students time to look at the two number patterns and read the sequences aloud together if required.

Ask “what is the same and what is different?” Give students time to talk to a partner or record ideas.

Ask students to share ideas. Record a list of similarities and a list of differences. Examples of similarities: both sequences include halves, all the numbers in pattern B appear in pattern A, all numbers are less than 20. Examples of differences: one sequence is counting forwards/ one is counting backwards, not all numbers in pattern A appear in pattern B, one sequence is counting in  $\frac{1}{2}$ 's and the other is counting in wholes.

To extend the task ask students to create a third number pattern that has at least one similarity and one difference to A and B.

### Other Examples

- A) 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8, 8.5, 9, 9.5 ...  
B) 13.5, 12, 10.5, 9, 7.5, 6, 4.5, 3, 1.5 ...
- A) 100, 101, 101, 102, 103, 103, 104, 105...  
B) 1000, 999, 999, 999, 998, 998, 998, 997 ...

## Curriculum Links

**During Year 4, 5 and 6**  
*Recognise and describe the rule for a growing pattern using words, tables, and diagrams, and make conjectures about further elements in the pattern*

## Big Ideas

*Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule. Patterns exist both in the world and in mathematics.*

## Suggested Learning Outcomes

*Notice similarities and differences between number patterns*

*Count forwards and backwards in whole numbers and fractions*

## Mathematical Language

Forwards, backwards, half, whole, similarity, difference, pattern

# Number Sequences on a number line

Here are two numbers in an arithmetic sequence.  
What might the sequence be?  
Can you continue the sequence on a number line?



## Teacher Notes

This activity address both adding/ subtracting decimals on a number line and identifying a rule for a linear pattern.

### Instructions:

Display the task. Allow students time to turn and talk about “what might the sequence be?”.

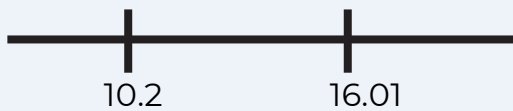
Discuss student's ideas. Whilst the most apparent rule for this pattern is  $+0.6$ , students might see this as  $-0.6$ , or have another valid idea.

Recap adding forwards or counting back in tenths to find the difference between 2.1 and 2.7 if needed.

Allow time for students to continue and record the sequence on a number line.

To generalise ask “if the rule is still  $+0.6$  and we start with 2.3 what would the new sequence be?”

## Other Examples



## Curriculum Links

### During Year 4, 5 and 6

*Add and subtract whole numbers and decimals to two places*

*Develop a rule in words about a linear pattern*

## Big Ideas

*A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.*

*Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule.*

*Patterns exist both in the world and in mathematics.*

## Suggested Learning Outcomes

*Explain a pattern in words and numbers*

*Continue a pattern*

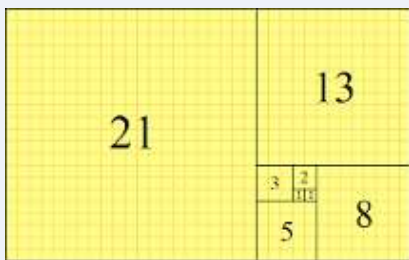
*Represent a pattern using a number line*

## Mathematical Language

*Tenths, hundredths, whole numbers, decimal numbers, halfway, between, sequence, pattern, rule*



# Exploring Fibonacci Sequence



What do you notice or wonder about this image?  
Can you find any patterns or relationships?

## Teacher Notes

This activity provides students the opportunity to begin exploring the Fibonacci sequence. In the Fibonacci sequence each number is the sum of the previous two numbers. E.g.,  $3 + 5 = 8$ ,  $5 + 8 = 13$  ...

### Instructions:

Display the task. Allow students time to turn and talk about what they notice/ wonder.

Discuss/share/record all the different aspects students notice or wonder about this pattern.

Students may notice that this pattern is growing (non-linear), previous numbers add to make the next number, the diagram uses square units to represent the number (e.g., 8 is written on an  $8 \times 8$  array).

You might prompt students to work out what the next numbers in the sequence will be, or predict what the 20<sup>th</sup> number might be, or to represent this pattern on a graph.

## Other Examples

Explore different representations of the same sequence. This sequence appears with nature (e.g., plant growth) and used by artists.

## Curriculum Links

### During Year 4, 5 and 6

Use tables, XY graphs, and diagrams to find relationships between elements of growing patterns.

Use a rule to make predictions.

## Big Ideas

Patterns are sequences (repeating or growing) made of numeric or spatial elements governed by a rule.

Patterns exist both in the world and in mathematics.

## Suggested Learning Outcomes

Explain a pattern in words and numbers

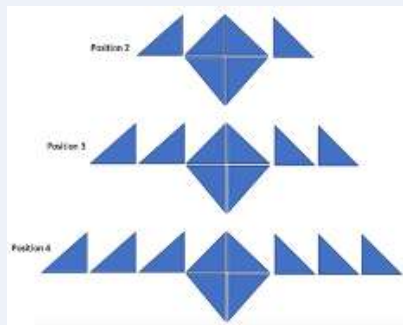
Make statements about what they notice and wonder

Predict further positions in a pattern

## Mathematical Language

Pattern, constant, changing, growth, relationship

# Finding Missing Positions



Look at this pattern. What is staying the same and what is changing?

What would position 1, 6 and 21 look like?

## Teacher Notes

### Instructions:

Show students the pattern. Discuss which positions are being shown. Ask students to turn and talk to the person next to them about what they notice.

Encourage students to notice what is staying the same (constant) and what is changing (variable).

Different students may “see” the pattern in varying ways, these different ways of explaining the pattern should be validated.

Ask students to describe what position 1 will look like using words.

Ask students to make a representation of position 6 (drawing, table, using materials etc). Share these different representations.

Discuss what rule we could use to find out what any position in the pattern will look like.

Develop a rule in natural language (words). E.g., “there will always be 4 triangles in the middle ...”, and model how we can record this in algebraic notation (e.g.  $2(x-1) + 4 = y$ )

## Other Examples

Present part of any geometric pattern (these can be easily found online). Continue to focus on describing what is staying the same and what is changing. You can extend the task by asking students to predict far positions. E.g., what would position 200 look like?



## Curriculum Links

### During Year 4

Recognise and describe the rule for a growing pattern using words, tables, and diagrams, and make conjectures about further elements in the pattern

### During Year 5 and 6

Use tables to recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule for the pattern in words, and make conjectures about further elements or terms in the pattern

## Big Ideas

Patterns can be made of numeric or spatial elements in a sequence governed by a rule.

A variable can be used to represent any number. Linear patterns and functions have a constant rate of change. They can be represented by ordered pairs, tables, XY graphs, and a rule (equation).

## Suggested Learning Outcomes

Explain and justify the pattern in relation to ordinal aspects of counting.

Identify the element for a repeating pattern for far terms.

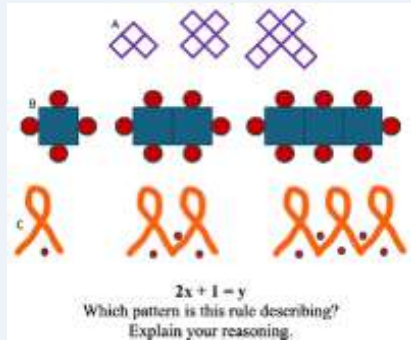
Explain that a pattern has consistency.

Develop generalisations for a repeating pattern and express it in words.

## Mathematical Language

Sequence, element, rule, unit of repeat, position, growth, constant

# Match the Pattern to the Rule



## Teacher Notes

### Key Concepts

$2x + 1 = y$  is a linear function.

$x$  is an independent variable (that changes according to the position number)  $+1$  is a constant (always present in any position)  $y$  is a dependent variable (it will change depending on the value of  $x$ ).

### Instructions:

Display the image. Give sufficient time for students to talk/ explore/record which pattern matches the rule.

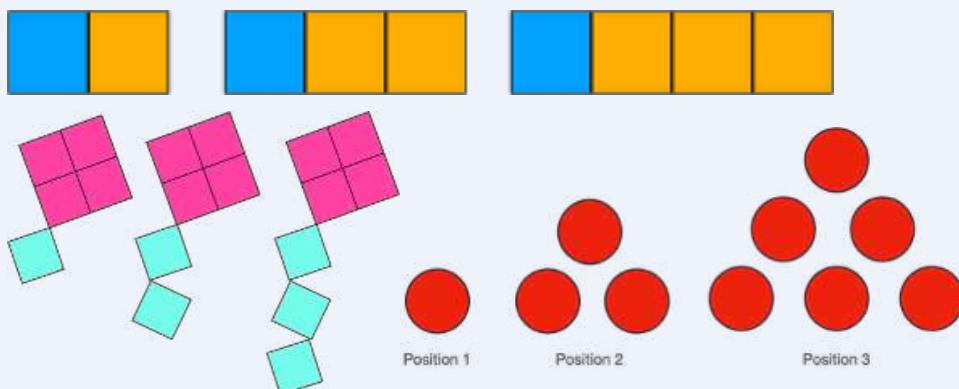
Listen for students who are talking about what is staying the same (constant) and what is changing (variable).

Facilitate a discussion on which pattern matches the rule and why/ along with why the other two do not match.

Encourage students to make the link that  $x$  represents the position number and  $y$  represents the total number of squares.

To extend the activity ask students to create a rule for the other two patterns. Or use the rule  $2x + 1 = y$  to find the number of squares in far positions such as 200 or 1000 etc.

## Other Examples



Which of these patterns shows the rule:  $x + 4$ . Discuss.

## Curriculum Links

### During Year 4

Recognise and describe the rule for a growing pattern using words, tables, and diagrams, and make conjectures about further elements in the pattern

### During Year 5 and 6

Use tables to recognise the relationship between the ordinal position and its corresponding element in a growing pattern, develop a rule for the pattern in words, and make conjectures about further elements or terms in the pattern

## Big Ideas

Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations. In a pattern, the relationship between the ordinal position (e.g., first, second, and third) and the corresponding element is more useful for finding the pattern's rule than the relationship between successive elements.

## Suggested Learning Outcomes

Explain that a pattern has consistency.

Develop generalisations for a repeating pattern and express it in words.

## Mathematical Language

Constant, unit of repeat, rule, sequence, variable

## This or that?

Complete the sentences using < > =

32cm \_\_\_\_ 302cm

3km \_\_\_\_ 2900m

1560cm \_\_\_\_ 15.6m

### Teacher Notes

In this activity, students decide which option is the greater amount (this or that). You can show one comparison at a time or all three at once.

Students can turn and talk and share their thinking with a partner. Encourage the use of agree or disagree and why with their partner.

Facilitate a large group discussion about different reasonings students had as they share their justifications.

Questions to support discussion:

How did you convert the units?

How could we work out what the difference is between the two?

What did you multiply/divide by to compare the measurements and why?

### Other Examples

Amounts can be changed to decimal and fractional numbers to increase challenge.

Length

500mm \_\_\_\_ 45cm

3cm \_\_\_\_ 28mm

55mm \_\_\_\_ 5cm

2000ml \_\_\_\_ 1l

3l \_\_\_\_ 1500ml

600ml \_\_\_\_ 0.5l

Capacity

500kg \_\_\_\_ 4800g

45000g \_\_\_\_ 405kg

380kg \_\_\_\_ 30800g

Gigabytes of Data

550mb \_\_\_\_ 0.5GB

10.5GB \_\_\_\_ 1500MB

28000MB \_\_\_\_ 2.45GB

### Curriculum Links

#### During Year 4

Use the metric measurement system to explore relationships between units

#### During Year 5

Use the metric measurement system to explore relationships between units, including relationships represented by benchmark fractions and decimals

#### During Year 6

Convert between common metric units for length, mass (weight), and capacity, and use decimals to express parts of wholes in measurements

### Big Ideas

When we measure, we use comparison, specifically, we compare like properties to see which is greater. We can make comparisons using standard or non standard units of measure and we use mathematical language to describe these.

Conceptual understanding of measurement requires understanding of conservation and transitivity.

### Suggested Learning Outcomes

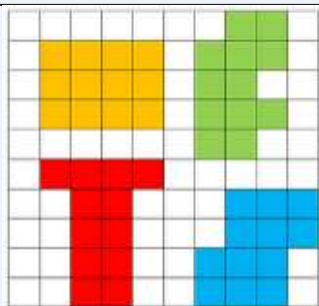
The metric measurement system is based on powers of ten.

Convert between units of measurement

### Mathematical Language

Unit of measure, measurement count, convert

## Area - Agree or Disagree



Tiana says that these shapes have a different area. Do you agree or disagree?

### Teacher Notes

Area:

- is measured in square units  $\text{cm}^2$
- is the space occupied by a 2-dimensional closed figure

Ensure when students are speaking about the figures, they use the language of “square units” and understand that measurement involves filling space.

### Instructions:

Give students time to discuss with a partner if they agree or disagree with the claim and why.

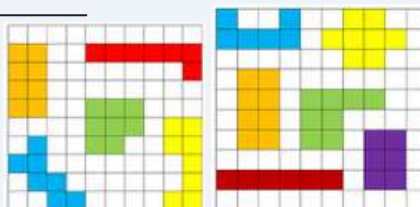
Share back ideas and notice which students hold a misconception (e.g., the shapes look different so the area must be different) and who can prove that all the shapes have the same area of  $12 \text{ square units.cm}^2$

Reinforce the idea that area involves finding out how many square units cover the surface of a figure.

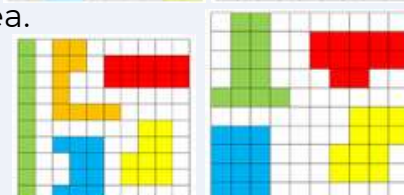
To extend the task ask questions such as: “will the perimeters of these shapes also be the same?” or provide students with  $1\text{cm}^2$  grid paper and see if they can make some more shapes that have an area of  $12\text{cm}^2$

### Other Examples

Shapes that have different areas:



Shapes that all have the same area.



### Curriculum Links

#### During Year 4, 5 and 6

Visualise, estimate and calculate the area of shapes covered with squares or partial squares.

### Big Ideas

Measures of area, volume, capacity and temperature can each be compared using ideas such as greater than, less than, and equal.

Measurement involves a selected attribute of an object (length, area, mass, volume, capacity) and a comparison of the object being measured against a unit of the same attribute.

### Suggested Learning Outcomes

Describe the area of a shape using square units

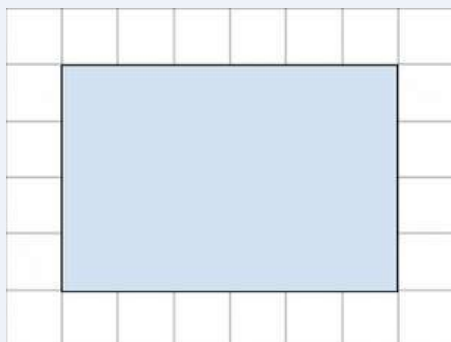
Agree or disagree with a statement using reasoning

### Mathematical Language

Area, rectangle,  $1\text{cm}$  square unit,  $\text{cm}^2$ , groups of, length, width, perimeter, greater than, less than



# Area of a Rectangle



**What is the area of this rectangle? How do you know?**

## Teacher Notes

Have multiplication charts available for students to access.

### Instructions:

Display image for 3 seconds (so students don't have time to count the squares 1:1).

Give a short amount of time for students to individually think about their estimate.

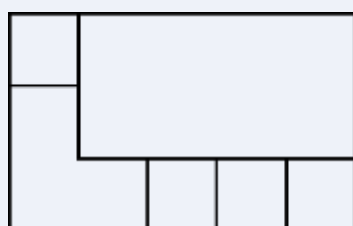
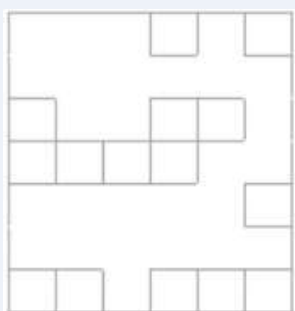
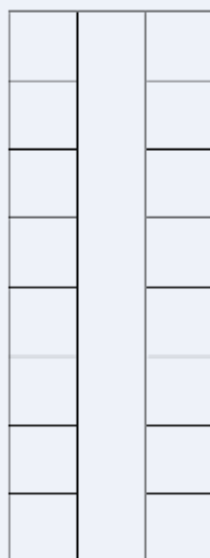
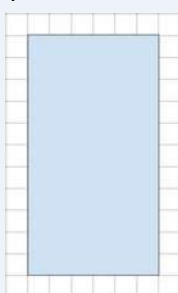
Show image again for 3 seconds.

Ask students to explain how they came up with their estimation. How did they see the squares? How many rows and columns? etc. Ensure students are using the type of unit in their explanations.

Show the image again (keep it displayed). Link student's explanations to the diagram and to multiplication.

## Other Examples

Different types of grids can be used including grids inside rectangles with missing squares.



## Curriculum Links

### During Year 4, 5 and 6

Visualise, estimate and calculate the area of shapes covered with squares or partial squares in  $\text{cm}^2$

## Big Ideas

Measures of area, volume, capacity and temperature can each be compared using ideas such as greater than, less than, and equal.

Measurement involves a selected attribute of an object (length, area, mass, volume, capacity) and a comparison of the object being measured against a unit of the same attribute.

## Suggested Learning Outcomes

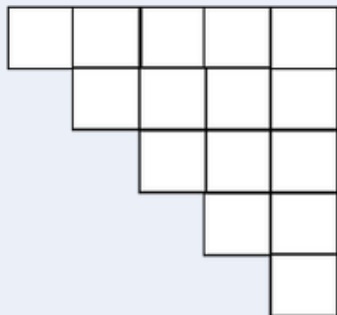
Describe the area of a shape using square units

Apply multiplication strategies to find the area of a rectangle

## Mathematical Language

Area, perimeter, unit of measure, measurement count, length, width, squares, squared

## Estimate and Find Perimeter of Shapes



**Sione puts two right triangles together to make a rectangle.  
Each of the triangles has an area of 15 square centimetres.  
What could be the perimeter of the rectangle?**

### Teacher Notes

*Use talk moves (turn and talk, adding to someone's idea, etc) to engage students with one another's noticings.*

*Get students to talk among themselves (in a group of 2-4 students), remember to start discussions with the open-ended question, "What do you notice?"*

*Give yourself space to listen to and can record students' noticings.*

*Look for an opportunity to take up one student idea and ask the whole class, "Why does that work?" or "How do you know?"*

### Other Examples

Sione puts two right triangles to make a rectangle. Each of the triangles has an area of 21 square centimetres. What could be the perimeter of the rectangle?

### Curriculum Links

#### **During Year 4, 5 and 6**

*Visualise, estimate and calculate the area of shapes covered with squares or partial squares in  $\text{cm}^2$*

### Big Ideas

For a given perimeter there can be a shape with area close to zero. The maximum area for a given perimeter and a given number of sides is the regular polygon with that number of sides.

### Suggested Learning Outcomes

Estimate and then measure length and area using appropriate metric units.

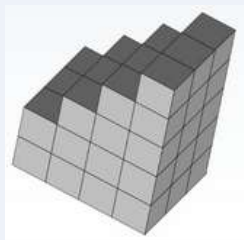
Visualise, estimate and find the perimeter and area of shapes composed of triangles and rectangles.

### Mathematical Language

Decomposed and composed shapes, metric units, estimate



# Visualising Volume



**How many blocks are there on the bottom layer of the tower?  
How many blocks would you need to complete the  
rectangular prism?**

## Teacher Notes

### Instructions:

*Launch the question one at a time to the students.*

*Give students time to talk to a partner about how they have calculated the number of blocks and volume,*

*Notice students who prove that the base layer has 12 blocks using multiplication/grouping strategies.*

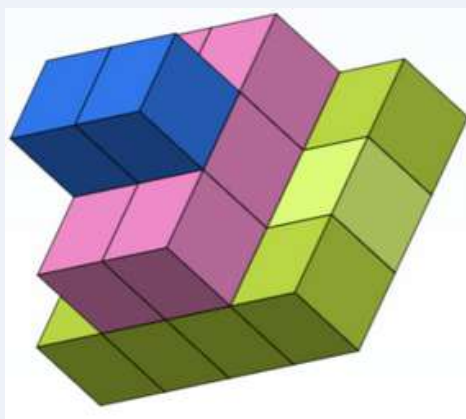
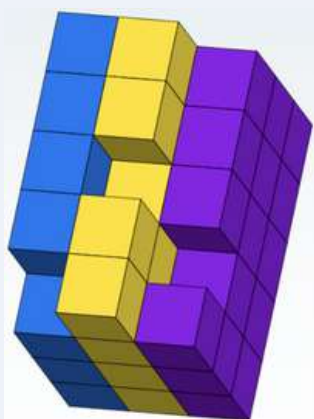
*Encourage the use of multiplication/grouping strategies to solve the second question. E.g. You would need one group of three blocks for each missing row. There are six missing rows.*

*Discuss the conceptual understanding that volume is the measurement of a 3D space therefore we need a 3D measuring tool (a cube) to measure the space. Make links to 3 cubes that have 1cm dimensions is  $1\text{cm}^3$ . These cubes can be layered to fill the space and we can use multiplication to work out how many cubic units there are on each layer.*

## Other Examples

Isometric drawings can be created on the following website:

<https://www.nctm.org/Classroom-Resources/Illuminations/Interactives/Isometric-Drawing-Tool/>



## Curriculum Links

### During Year 4 and 5

*Visualise, estimate and calculate the volume of rectangular prisms filled with centicubes, taking note of layers and stacking.*

### During Year 6

*Visualise, estimate and calculate the volume of rectangular prisms (in  $\text{cm}^3$ )*

## Big Ideas

*Measurement involves a selected attribute of an object (length, area, mass, volume, capacity) and a comparison of the object being measured against a unit of the same attribute.*

## Suggested Learning Outcomes

*Estimate the volume of a cuboid*

*Calculate volume using cubic centimeters*

## Mathematical Language

*volume, rectangle, 1cm cubic unit,  $\text{cm}^3$ , groups of, length, width, height*

# Time and Angles



What other times can you find where the clock hands form a straight angle?

## Teacher Notes

### Instructions:

Show the clock image to the students.

What do they notice about the position of the hands?

Discuss 180 degrees/straight angle.

Clarify with the students that this could be either 6am or 6pm. Therefore, in a 24-hour day, we have already found two examples of times that the minute and hour hands create a 180-degree angle.

Get the students to work in pairs or small groups to find other times that have the hands at a straight angle/180 degree (you may need to have printed outlines, mini clocks or access to an online tool).

Record the times they have found. Ensure students explain their times using the language of hours and minutes.

Address any misconceptions that arise, such as time is measured in base-60, or each number on the clock can represent either the hour or an interval of 5 minutes.

## Other Examples

What times can you find where the hands make a 90-degree angle? / 45-degree angle? / 270-degree angle? Less than 10-degree angle?

## Curriculum Links

### During Year 4

Tell time to the nearest 5 minutes, using the language of 'minutes past the hour' and 'to the hour'

Recognise that angles can be measured in degrees, using 90, 180 and 360 degrees as benchmarks

### During Year 5 and 6

Describe angles using the terms acute, obtuse, straight and reflex, comparing them with benchmarks of 90, 180 and 360 degrees

## Big Ideas

A clock is a circular number line – the hands move gradually around this number line.

On an analogue clock the hour hand shows the approximate time in the day and the minute hand shows a more exact time.

## Suggested Learning Outcomes

Recognise and describe angles of 180 degrees

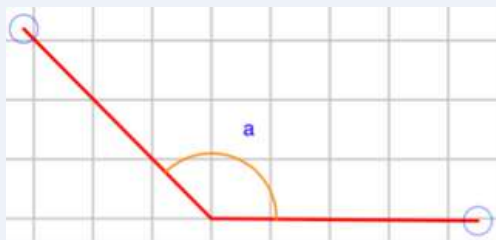
Read and write times on an analogue clock

Convert between units of time

## Mathematical Language

Number words, clock, half past, past, to, o'clock, angle, 180 degrees, 24 hours, 12 hours, hands, minutes, hours.

# Estimating Angles



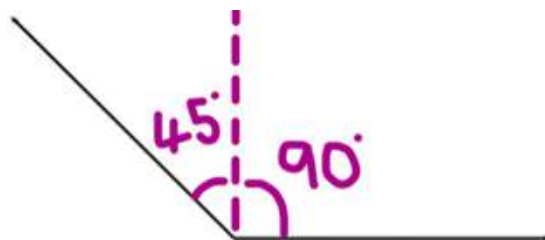
**What is the size of this angle? Estimate.**

## Teacher Notes

### Instructions:

Display the image and give time for students to turn and talk about the size of the angle.

Share ideas that draw on the use of benchmarks and annotate so these are clear. E.g.,

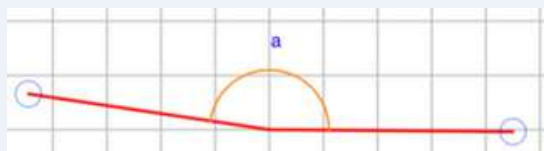
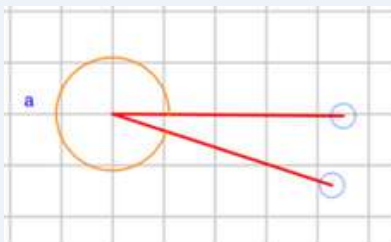
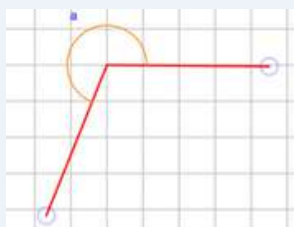


Students might notice  $90^\circ + 45^\circ$ , or  $180^\circ - 45^\circ$ . Reinforce the understanding that  $45^\circ$  is half of  $90^\circ$ ,  $90^\circ$  is half of  $180^\circ$ ,  $180^\circ$  is half of  $360^\circ$

To extend the task ask students “what angle will we need to add up to  $360^\circ$ ?” E.g.,  $135^\circ + ? = 360^\circ$   
Repeat with a range of other examples, increasing the complexity over time.

## Other Examples

Online tools to make angles are easily found online; such as <https://www.visnos.com/demos/basic-angles>



## Curriculum Links

### During Year 4

Recognise that angles can be measured in degrees, using  $90$ ,  $180$  and  $360$  degrees as benchmarks

### During Year 5

Describe angles using the terms acute, obtuse, straight and reflex, comparing them with benchmarks of  $90$ ,  $180$  and  $360$  degrees

### During Year 6

Visualise, measure, and draw (to the nearest degree) the amount of turn in angles up to  $360$  degrees

## Big Ideas

Angles can be compared using ideas such as greater than, less than, and equal. A number of degrees can be used to describe the size of an angle's opening.

## Suggested Learning Outcomes

Recognise and use benchmarks of  $45^\circ$ ,  $90^\circ$ ,  $180^\circ$

Describe angles as being greater or small than a benchmark

Estimate using mathematical reasoning

## Mathematical Language

Angle, greater than, less than,  $45^\circ$ ,  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ ,  $360^\circ$

# Time: 12-Hour vs 24-Hour

**Part 1: What is the same and what is different about these two times?**



**Part 2: Put the following times in order. Explain and justify your decisions.**



## Teacher Notes

### Instructions:

**Part 1:** Give students time to turn and talk about the first image. Listen for student reasoning that draws on understandings of 12 vs 24-hour time.

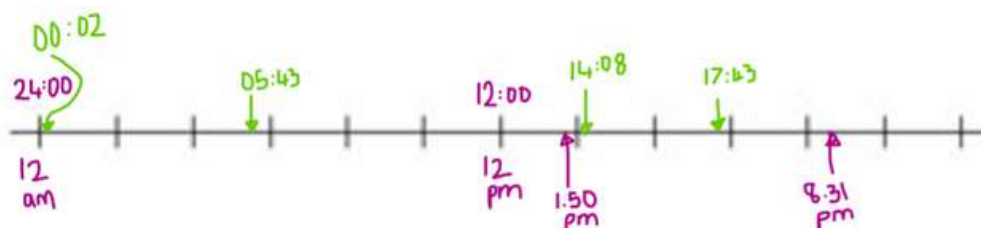
Reinforce the idea that 12-hour clock goes up to 12:00pm (midday), then repeats a second cycle, ending at 12:00am (midnight). We use of am or pm to determine which part of the day we are talking about. The 24-hour digital clock begins at 00:00 (midnight) and 12:00pm (midday) is shown as 12:00, 1:00pm is 13:00, 2:00pm is 02:00, etc. We do not use am or pm for 24-hour time, as the value of the digits identify which part of the day the time relates to.

**Part 2:** Ask students to order the 6 times from earliest in the day to latest. You might print out cards to manipulate, or prompt students to order the times on a number line.

Listen for/ highlight student reasoning that uses conversion between 12- and 24-hour time or draws on benchmarks such as midday, midnight.

Clearly represent the order using a visual timeline. E.g.,

Ask students to discuss where they have seen 24-hour time used, and why this might be useful.



## Curriculum Links

### During Year 4

Tell time to the nearest 5 minutes, using the language of 'minutes past the hour' and 'to the hour'

### During Year 5

Describe the difference in duration between units of time (e.g., days and weeks, months and years), and solve duration-of-time problems involving 'am' and 'pm' notation

### During Year 6

Convert between units of time and solve duration-of-time problems, in both 12- and 24-hour time systems

## Big Ideas

Time is displayed in different ways depending on the context. Time measurements can be compared when they are converted into the same unit.

## Suggested Learning Outcomes

Read 12- and 24-hour digital clocks

Convert 24-hour time to 12-hour time

Order times from earliest to latest in a day

## Mathematical Language

Time, hours, minutes, seconds, 12-hour clock, 24-hour clock, AM, PM, midday, midnight, earlier, later

## Other Examples

Are these the same/ or different times? Explain why.

1.



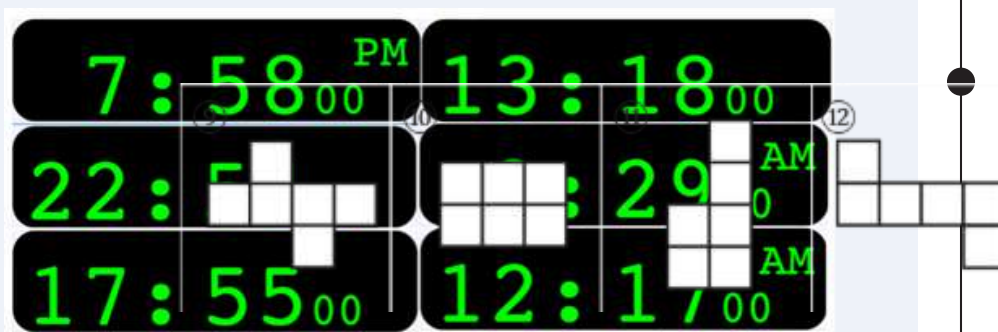
2.



3.



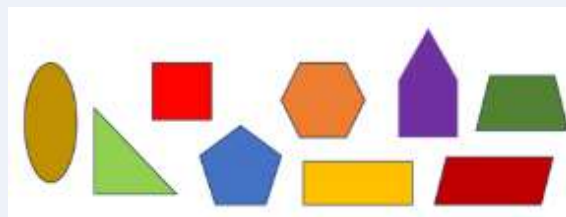
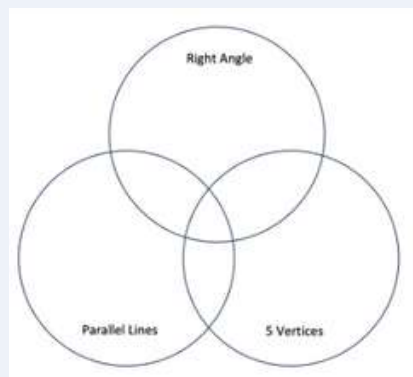
Put the following times in order.  
Explain and justify your decisions.



This activity could also be repeated using a mixture of both analogue and digital clocks.



# SORTING BY ATTRIBUTES



Where would each shape go and why?  
Materials - shapes and sorting circles

## Teacher Notes

*The purpose of this activity is for students to notice and explain relationships between shapes and their properties.*

*Right angle: exactly  $90^\circ$ , or a quarter turn*

*Vertices: the point where 2 or more lines meet (corner)*

*Parallel lines: two lines that are same distance from each other and never meet*

*Pentagon: a 5-sided shape that has 5 straight edges, 5 vertices and 5 internal angles that add to  $540^\circ$*

### Instructions:

*Explain the purpose of a Venn diagram.*

*Students to choose a shape and place it within (or outside) the diagram.*

*Expect students to explain using geometric reasoning "this shape has a right angle and parallel lines because ..."*

*Allow other students the opportunity to agree or disagree with where the shape is placed.*

*Once the diagram is complete encourage students to make statements about what they notice. E.g., "There is one shape that has 5 vertices, parallel lines and a right angle", "there is one shape that does not fit inside the Venn diagram because..."*

*Use these findings to make claims about classes of shapes. E.g., some pentagons have a right angle, but not all.*

## Other Examples

Repeat the task but change the properties and set of shapes.

E.g., 6 vertices, curved line, non-parallel lines

## Curriculum Links

*Two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.*

## Big Ideas

*Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analysed by their attributes.*

*There is more than one way to classify most shapes and solids.*

## Suggested Learning Outcomes

- Visualise, identify, compare, and classify two- and three-dimensional shapes
- Identify relationships, including similarities, differences, and new connections
- Use geometrical language to describe shapes and objects.

## Mathematical Language

*Sides, vertices, parallel lines, right angle same, different, properties, shape names*

# Geometric Statements

## Are these statements: always, sometimes or never true?

- A square is a rectangle.
- A square is a rhombus.
- A rhombus is a rectangle.
- A parallelogram is a rectangle.
- A parallelogram is a trapezium.
- A kite is a trapezium.

## Teacher Notes

### Instructions

*Present each statement to the class one at a time*

*Allow time for students to discuss/record with a partner about whether the statement is always, sometimes or never true.*

*Facilitate a discussion about the statement. Push for explanations and justifications. Notice if students are reasoning using geometric properties (e.g., both squares and rectangles have 4 right angles).*

*Record the properties students discuss and use these to develop a working definition of each shape (that can be added to and refined over time as students' understandings grow).*

*Ask students to give examples if the statement is sometimes or never true/ or provide counterexamples.*

*Students could check their thinking against a geometric classification chart.*

## Other Examples

Use statements that will encourage students to think about classes of shape/ or properties they need to deepen their understandings about.

## Are these statements: always, sometimes or never true?

- The internal angles of a triangle equal  $180^\circ$
- A triangle has a right angle.
- A scalene triangle has 2 equal sides.
- A triangle has three vertices.
- A triangle is a polygon.
- A trapezium can be made from triangles.

## Are these statements: always, sometimes or never true?

- A hexagon has six equal length sides.
- Triangles have a line of symmetry.
- Squares have two diagonals that meet at right angles.
- The base of a pyramid is a square.
- A cuboid has two square faces.
- Quadrilaterals can be cut into two equal triangles.

## Curriculum Links

*Two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.*

## Big Ideas

*Two- and three-dimensional objects with or without curved surfaces can be described, classified, and analysed by their attributes. There is more than one way to classify most shapes and solids.*

## Suggested Learning Outcomes

- Identify the properties of shapes
- Classify shapes using geometric properties and reasoning

## Mathematical Language

Quadrilateral, square, rectangle, rhombus, parallelogram, trapezium, kite, sides, vertices, angles, right angles, parallel sides



# Angles in Shapes

What angles can you find in the Tongan and Philippines flags?



## Teacher Notes

### Instructions

Display the image and give time for students to turn and talk about the angles they can see. Listen for students who are noticing or comparing to benchmark angles (e.g. less than  $90^\circ$ )

Ask students to mark the angles on the image.

Students may find multiple examples of  $90^\circ$ , describe angles that are less than/great than  $90^\circ$ , or draw on their knowledge of shapes (e.g. in the Philippines flag the white triangle is equilateral, so it will have three  $60^\circ$  angles).

Compare the angles in the two flags, are they the same or different? Do you think all flags will contain the same angles?

## Other Examples

Repeat the task with other interesting flags, buildings, cultural artefacts or artwork suitable to your students. E.g.,



## Curriculum Links

- Describe an angle using the benchmarks 90 degrees, 180 degrees, and 360 degrees.
- Angles are a measure of turn and can be measured in degrees

## Big Ideas

Angles can be compared using ideas such as greater than, less than, and equal. A number of degrees can be used to describe the size of an angle's opening. There is more than one way to classify most shapes and solids.

## Suggested Learning Outcomes

- Recognise angles within shapes and common objects
- Describe angles as being greater or small than benchmark angles.
- Estimate using mathematical reasoning

## Mathematical Language

Angle, greater than, less than,  $45^\circ$ ,  $90^\circ$ ,  $180^\circ$ , right angle

# Symmetry

Write some words that have horizontal or vertical symmetry

## Teacher Notes

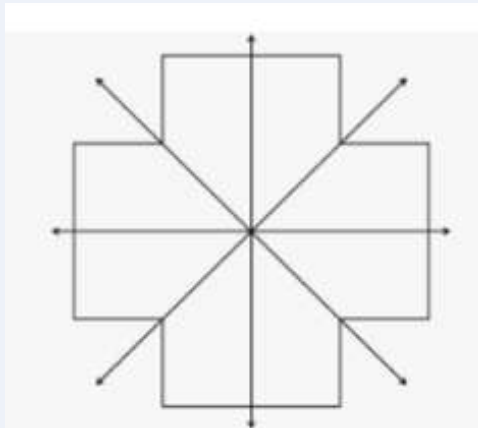
Students could first identify horizontal and vertical lines of symmetry in uppercase and lowercase letters.

Some examples **COOK** **mum**

## Other Examples

Draw shapes that have four lines of symmetry

Example



## Curriculum Links

- Two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.
- Shapes can be rotated, reflected, translated, and resized.

## Big Ideas

A transformation is a way of moving a shape, and a shape that remains unchanged under a transformation is said to have symmetry.

Transformations provide a significant way to think about the ways properties change or do not change when a shape is moved on a plane. Line symmetry is a component of the transformation called a reflection.

## Suggested Learning Outcomes

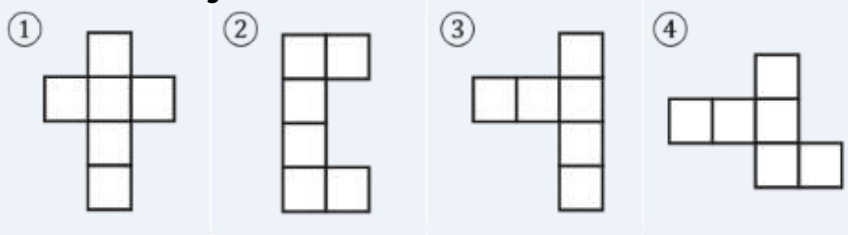
Perform and describe rotations, reflections, translations, and resizing on two-dimensional shapes and simple geometric patterns.

## Mathematical Language

Reflection, mirror line, mirror symmetry, reflectional symmetry, line of symmetry, flipping, congruent, transformation.

# Create Nets for a Cube

**How many of these nets can make a cube?**



Explain and justify how you know.

## Teacher Notes

Students can turn and discuss which nets make a cube.

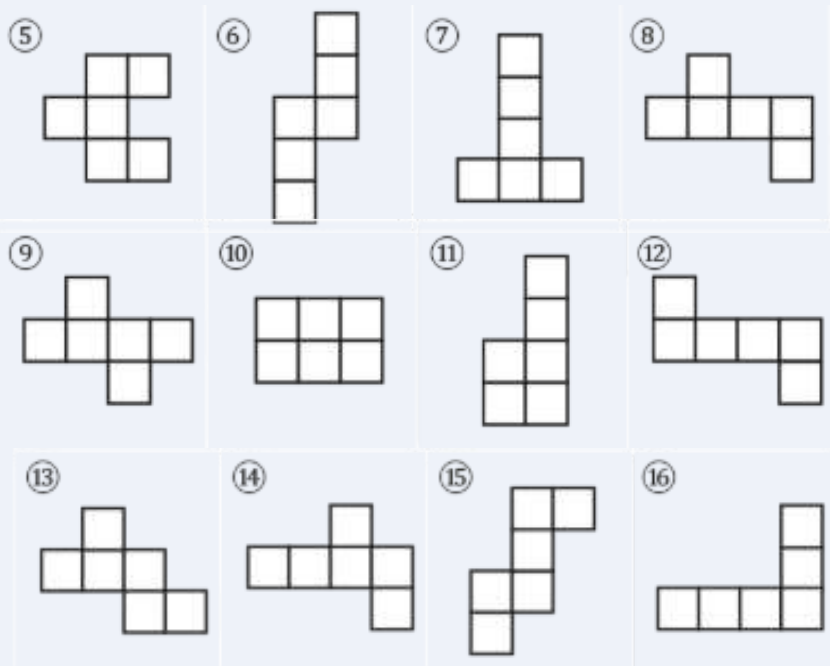
Teacher can also give students grid paper if it is available, if not students can use blank paper to create their nets. Also notice the students who use gesturing for the number of faces needed.

Have opportunities for them to cut out the nets to prove which ones will make a cube during Independent activities.

Compare the difference between a cube and cuboid: The key difference between a cube and a cuboid is: a cube has six square-shaped faces of the same size, but a cuboid has rectangular faces. A cuboid is also known as a rectangular prism.

## Other Examples

Here are some other examples you can use on other days to talk about nets of a cube.



## Curriculum Links

Two and three dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.

## Big Ideas

Two- and three-dimensional shapes have consistent properties that can be used to define, compare, classify, predict, and identify relationships between them.

## Suggested Learning Outcomes

- visualise and draw nets for rectangular prisms.
- classify two-dimensional shapes and prisms using their spatial properties to justify my classifications

## Mathematical Language

cube, cuboid, rectangular prism, net, Properties, square, attribute, 2-dimensional, 3-dimensional, shape, side, equal, size, straight, parallel, congruent, quadrilateral, faces, edges, vertices,

# Create Nets for a Cube

---

## Other Examples

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You can also add in a net that has more than six faces or that are more rectangular to add more curiosity to the warm up.

### Extension

How many different nets can you draw for a cube?

- Teacher can write this question on the board and allow students to explore and create nets.
- Teacher can show students a picture of a cube or bring in an object shaped like a cube eg present box.
- Students can be given paper to draw as many nets as they can think of for a cube.
- Tell the students not to draw the flaps just the faces.

This activity will need to be repeated so that they have the opportunity to get closer and closer to drawing the net. As they complete an iteration have them open the box and compare their nets.

- Also notice the students who use gesturing for the number of faces needed.
- Have opportunities for them to cut out their nets to compare theirs nets for a cube.

# Viewpoint on Maps



## Teacher Notes

*Cardinal Directions: N,S,E,W.*

*Ordinal Directions: NE,NW, SE,SW*

## Instructions

*Display the image and give time for students to look at the map and think about the landmarks they can see.*

*Ask the first question. Give students time to talk to a partner.*

*Listen for/ and highlight student responses that use the compass directions. E.g., “he will see the house because the compass shows this is North”. You might also like to draw this pathway on the map.*

*Repeat as above for each question. Continue to highlight responses that draw on directional reasoning. Discuss how to find ordinal directions (e.g., North-East) if this is new to students.*

## Other Examples

Choose any map that will be relatable or interesting for your students. Prepare questions that will encourage students to think about viewpoints from various locations or landmarks.

## Curriculum Links

*Use grid references, the language of direction (compass points), distance (in m, km), and turn (in degrees) to locate and describe positions and pathways.*

## Big Ideas

*Maps use grid references or coordinates to specify places, scales to show distances, and connections to show pathways.*

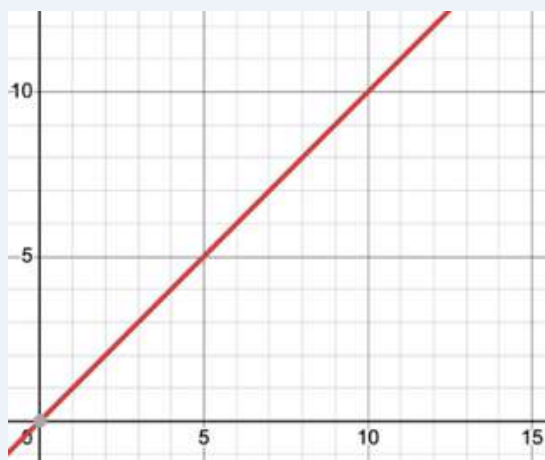
## Suggested Learning Outcomes

- Identify landmarks on maps
- Describe viewpoints from landmarks
- Use compass points (N,S,E,W) to describe pathways

## Mathematical Language

Landmark, compass, compass points (North, South-West etc), direction, viewpoint, pathway

# Exploring Linear Graphs



## Teacher Notes

*The purpose of this task is for students to explore and discuss linear graphs.*

### Instructions

*Ask students “what do you notice about this graph”*

*Give time for students to discuss with a partner.*

*Share back student's ideas and annotate/record on the graph.*

*Direct students to the x and y axis. Discuss the relationship between the x and y axis. E.g,  $5x = 5$ .  $10x = 10$*

*Ask students to describe the slope of the line in words.*

*Ask questions that will extend students thinking. E.g., “why do you think the line goes below zero?”, “how far might this line continue for?”, “where will the line for  $15x$  be?”*

## Other Examples

Create your own graph online e.g.,  
<https://www.desmos.com/calculator>

Explore simple functions such as  $2x=y$ ,  $3x=y$ ,  $4x=y$  ... and compare

## Curriculum Links

- Use tables, XY graphs, and diagrams to find relationships between elements of growing patterns.
- Develop a rule in words about a linear pattern.
- Use a rule to make predictions.

## Big Ideas

*Linear patterns and functions have a constant rate of change. They can be represented by ordered pairs, tables, XY graphs, and a rule (equation).*

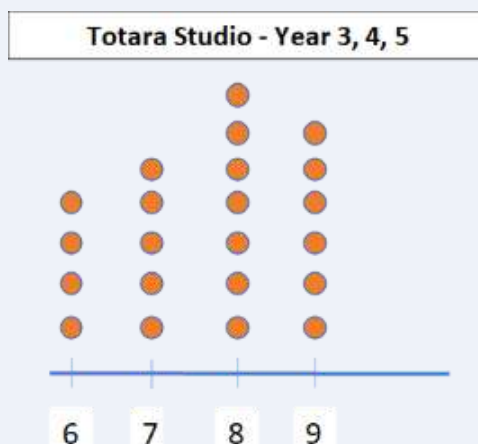
## Suggested Learning Outcomes

- Make a statement about what they notice
- Describe the slope of a graph in words or numbers

## Mathematical Language

Graph, axis, linear function, constant, variable, rule, slope.

# Dot Plot



What could this graph be telling us? Explain your reasoning.

## Teacher Notes

### Instructions

Show students the dot plot and provide a copy per pair

Facilitate the students to make connections to their investigative question.

Monitor for students using the vocabulary of statistics.

Particularly focus on students making comparative statements in relation to the data.

Facilitate students to give reasons for statements and ask if they agree or disagree

## Other Examples

Show students different types of graphs so that they become familiar with them.

## Curriculum Links

- Use and describe a variety of data visualisations, identifying features, patterns, and trends in context and answering the investigative question
- Interrogate others' survey or data-collection questions, and identify and explain features and errors in others' data visualisations and statements about data.

## Big Ideas

Data can vary in different ways (e.g., an object can be different sizes and colours) and it can be organised in different ways and by different characteristics (categorical, numerical) Data can be represented and communicated in multiple ways including data visualisations Patterns can be noticed, described, and analysed in sets of data and by using data visualisations

## Suggested Learning Outcomes

- Agree or disagree with statements about data displayed on a graph.
- Provide reasons and evidence for statements about data displayed on a graph.
- Make statements about data displayed on a graph.

## Mathematical Language

Statistics, data, sample, investigate, organise, display, sort, classify, represent, communicate, predict, outcomes, compare, similarities, differences, tally chart, graph, dot plot.



# What could these graphs be telling us about?



What could these different graphs be telling us about? Explain your reasoning.

## Teacher Notes

*Different types of graphs have different purposes. The choice of graph will support the effective and clear display of data. E.g., Pie graph: useful for showing percentages of a whole, represents a set point in time*

*Bar graph: useful for comparing categorical data, allows easy comparison of the size of different groups. Each bar represents a categorical variable, allows comparison of the size of different sets/groups*

*Line graph: useful for showing information that changes over time.*

### Instructions

*Show students the graphs (could be one at a time or all together)*

*Let them discuss in groups/pairs what the graphs could be about and why.*

*Be listening for the statistical language students know and use (add to a statistics vocab wall). What knowledge of different graphs do students have? Are they aware that data displays have different purposes?*

*Ask students to share their ideas. Agree/ or disagree if a particular data set could be shown a particular graph and why. (E.g., "what is your favourite sport?" is not appropriate for the line graph.*

*Ask them how they could interpret the data shown (E.g.; The blue segment of the pie graph shows the most, it is over half. There are four different categories shown on the bar graph)*

## Other Examples

Show students different types of graphs so that they become familiar with them.

## Curriculum Links

- Use and describe a variety of data visualisations, identifying features, patterns, and trends in context and answering the investigative question
- Interrogate others' survey or data-collection questions, and identify and explain features and errors in others' data visualisations and statements about data.

## Big Ideas

*Data can vary in different ways (e.g., an object can be different sizes and colours) and it can be organised in different ways and by different characteristics (categorical, numerical) Data can be represented and communicated in multiple ways including data visualisations Patterns can be noticed, described, and analysed in sets of data and by using data visualisations*

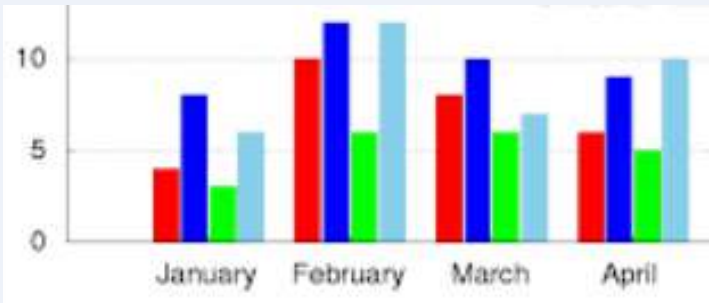
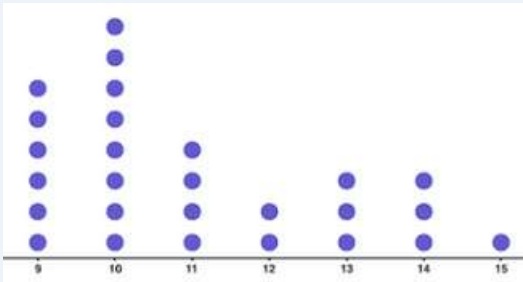
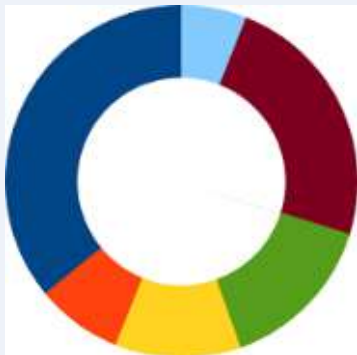
## Suggested Learning Outcomes

- Agree or disagree with statements about data displayed on a graph.
- Provide reasons and evidence for statements about data displayed on a graph.
- Make statements about data displayed on a graph.

# What could these graphs be telling us about?

## Other Examples

Explore a wide variety of different un/ or partially labelled displays and graph types. Continue to reinforce why certain graphs are more effective for displaying certain data sets.



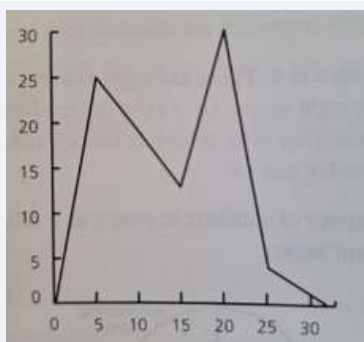
	5	9, 9
6, 8, 9	6	6, 7, 8, 8, 9, 9
1, 2, 3, 5, 5, 6, 8	7	0, 2, 2, 5, 7, 7, 8
0, 1, 2, 3, 4	8	

## Curriculum Links

### Mathematical Language

Statistics, data, sample, investigate, organise, display, sort, classify, represent, communicate, predict, outcomes, compare, similarities, differences, tally chart, graph, dot plot.

# Interpreting Graphs



This is the number of children talking in class over a period of 30 minutes. What time of day might this line graph represent?

## Teacher Notes

*Give the students time to discuss the numbers of the graphs.*

*Guide them to discuss which axis might represent the frequency and which axis represents the number of minutes*

*Break down what is happening at each point. How many students are talking at the five minute mark?*

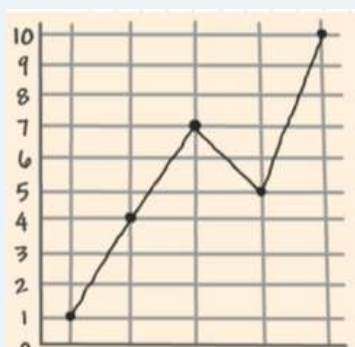
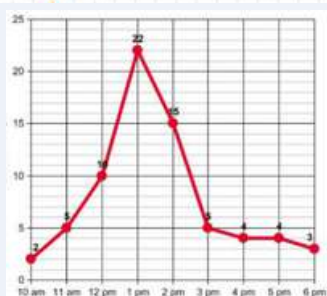
*How many students are talking at the fifteen minute mark?*

*Why might all the students be talking at the 20 minute mark?*

*What might be happening between the 5 minute mark and the 15 minute mark?*

## Other Examples

What might these line graphs be representing? Label the axis and justify your decisions.



## Curriculum Links

- Use and describe a variety of data visualisations, identifying features, patterns, and trends in context and answering the investigative question
- Different data visualisations for the same data can lead to different insights.

## Big Ideas

*Ideas and questions about a specific topic can be investigated through collecting data and using it to answer the questions. Data can vary in different ways (e.g., an object can be different sizes and colours) and it can be organised in different ways and by different characteristics (categorical, numerical).*

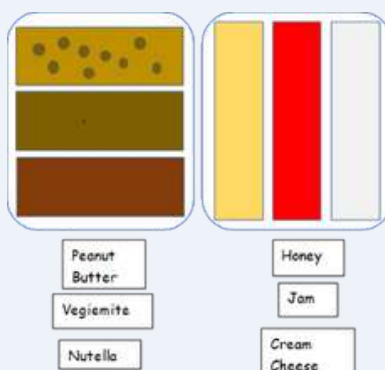
## Suggested Learning Outcomes

- use and describe a variety of data visualisations, identifying features, patterns, and trends in context and making connections to the group of interest
- justify their reasoning making links to the graph

## Mathematical Language

Statistics, data, sample, investigate, organise, display, sort, classify, represent, communicate, predict, outcomes, compare, similarities, differences, line graph.

# Probability – Quick think



If these 2 slices of bread were to be made into a sandwich, how many combinations of tastes would there be on the sandwich?

## Teacher Notes

*Provide the pictures of the different options to the students.*

*Notice whether students can systematically record the different options and work out how many different combinations are possible.*

*During the discussion the possibility of each option could be linked to fractions in relation to the chance of each combination.*

*During the discussion remind students of a tree diagram to record different combinations.*

## Other Examples

How many different combinations of flavours can you have on your shaved ice cone?



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E.g.: white icing + chocolate chips + yellow dots + Sprinkles + mini marshmallows etc.

Repeat for chocolate icing, blue icing, yellow icing, orange icing etc.

## Curriculum Links

- pose investigative questions for a chance-based situation with equally likely outcomes, listing all possible outcomes for the situation
- compare my findings with those of others when undertaking probability experiments.

## Big Ideas

*The world is characterised by change and variation that we use mathematics and statistics to understand.*

*A probability experiment involves repeated trials.*

*Results may vary in trials.*

*The experimental probability of an event is the number of times the event occurs divided by the total number of trials.*

## Suggested Learning Outcomes

- Represent the different outcomes for an event.
- Find all of the possible outcomes for an event.

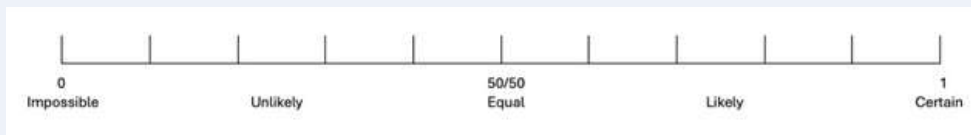
## Mathematical Language

Combinations, Probability, chance, unlikely, possible, likely, certain, equal, chance.

# Likelihood Line

Where would you place each event on the likelihood line?

1. You will ride a bike today
2. It will rain after lunch
3. A goat will come into the classroom
4. The sun will set tonight
5. You will eat fruit at lunchtime
6. You will go swimming this weekend



## Teacher Notes

*Display the continuum on the TV screen, board or provide printed copies among small groups.*

*Explain the continuum as a scale from 0-1, impossible to certain which describes the possibility of an event occurring.*

*Have children discuss with a buddy the events listed and which probability most closely matches their thoughts on whether it will occur.*

*Encourage children to provide reasons that support their statement using the word “because”. For instance, “It is likely I will ride my bike today after school because I don’t have any sports practice and will have time”. Consider how some statements can be argued, for instance “It is unlikely we will have fish and chips for dinner because it’s only Tuesday, if it were Friday the chance would be highly likely”.*

*Extend students to include fractions, decimals and percentages to match the probability language, eg. .5, 50% or  $\frac{1}{2}$  match equal chance.*

*Add these labels to the continuum over several days to create a co-constructed resource for your maths wall.*

## Other Examples

- You will need to wear a hat in the playground during break time
- You will eat fish and chips for dinner
- Mum will cook your favourite meal this weekend
- Blue house will have the most points at the end of term.

Consider other scenarios that could be added to the list, including ones that are relevant to your learners/school.

## Curriculum Links

*Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain).*

## Big Ideas

*The chance of an event occurring can be described numerically by a number between 0 and 1 inclusive and used to make predictions about other events.*

## Suggested Learning Outcomes

- Make statements about the likelihood of an event happening.
- Justify the placement of events of the likelihood line.

## Mathematical Language

Combinations, Probability, chance, unlikely, possible, likely, certain, equal, chance.



# Comparing Results

What do you notice? What do you wonder?  
What is your prediction for the results of Day 4?

Color	Count	Experimental %
Blue	8	32.0%
Yellow	6	24.0%
Cyan	2	8.0%
Red	9	36.0%

Day 1

Color	Count	Experimental %
Blue	7	28.0%
Yellow	12	48.0%
Cyan	6	24.0%
Red	0	0.0%

Day 2

Color	Count	Experimental %
Blue	5	20.0%
Yellow	8	32.0%
Cyan	6	24.0%
Red	6	24.0%

Day 3

## Teacher Notes

The focus of this activity is for students to understand that the results of trials will differ each time, and these results may/or may not reflect the theoretical probability. The larger number of trials conducted, the more likely the results should reflect the theoretical probability.

### Instructions

Display the image and give students time to turn and talk about what they notice and what they wonder.

Facilitate a group discussion on what students notice and wonder. Record/annotate these ideas and ask questions that will deepen student thinking.

Several mathematical ideas could be reinforced here (e.g., percentages for each trial must add to 100%, the difference between experimental and theoretical probability, the range of results each day, or how to read and interpret a graph).

Ask students to predict what might happen on Day 4. Notice if students have flawed logic (e.g., look at previous independent trials to make a prediction) or if they realise each set of trials is an independent event not influenced by previous events.

## Curriculum Links

- Probabilities and the language of probability are associated with values between 0 or 0% (impossible) and 1 or 100% (certain).
- A probability experiment involves repeated trials. Results may vary in trials.

## Big Ideas

The world is characterised by change and variation that we use mathematics and statistics to understand. A probability experiment involves repeated trials. Results may vary in trials. The experimental probability of an event is the number of times the event occurs divided by the total number of trials.

## Suggested Learning Outcomes

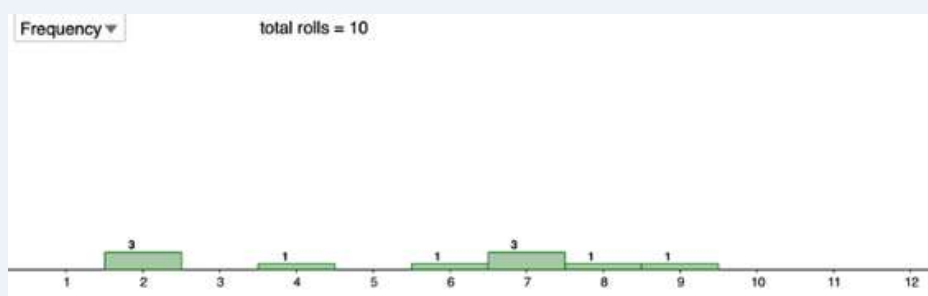
- Identify similarities and differences in results of trials
- Compare theoretical and experimental probabilities
- Make statements and form questions about trial results

## Mathematical Language

Trial, outcomes, sample size, theoretical probability, experimental probability, similar, different, percentage

## Other Examples

These graphs show the sum of two dice after 10, 100 and 1000 trials. What do you notice? What do you wonder?



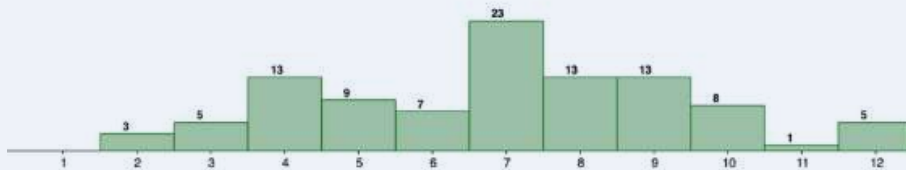
# Comparing Results

## Other Examples

These graphs show the sum of two dice after 10, 100 and 1000 trials. What do you notice? What do you wonder?

Frequency ▾

total rolls = 100



Frequency ▾

total rolls = 10000

